Risk Sharing and Risk Reduction with Moral Hazard*

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Abstract

This paper studies the design of a Financial Stability Fund under different incentive provisions. We generalize the *flexible* moral hazard framework of Georgiadis et al. (2024), where an agent freely chooses next period's shock distribution, to a dynamic recursive contract. Unlike the canonical model of Atkeson and Lucas (1992), this approach provides incentives based on rewarding the agent's marginal costs rather than realized outcomes. The optimal contract features *bliss*, as opposed to *immiseration*, and incentive provision does not disrupt risk-sharing; in fact, risk reduction is optimal. While neither model nests the other, we make them comparable: restricting distributions in the flexible model and back-loading incentives in the canonical one. We provide a quantitative analysis for Euro Area stressed countries, comparing the implications of each moral hazard framework. These results offer insights for the design of official lending programs when countries differ in their ability to manage endogenous risks.

Keywords: Official lending, limited enforcement, moral hazard, risk sharing, recursive contract

JEL classification: E43, E44, E47, E62, F34, F36, F37

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1 Introduction

In models of debt and risk-sharing, moral hazard (MH) is a central concern: a risk-averse borrower can improve its risk profile through effort, but since effort is non-contractible, lenders have to provide *the right incentives for* the borrower to make *the right effort*. Mechanism design achieves this incorporating incentive compatibility (IC) constraints in the design of constrained efficient debt-and-insurance contracts. However, the introduction of IC constraints is not uniform across theoretical frameworks, leading to different constrained efficient outcomes. Furthermore, while the inclusion of IC constraints in debt and risk-sharing models is well-understood, how these constraints interact with risk-sharing and limited enforcement (LE) constraints is not.

This paper addresses these issues contrasting two existing moral hazard theoretical frameworks – the well-established *canonical MH* framework pioneered by Holmstrom (1979) and in dynamic contracts by Atkeson and Lucas (1992), and the new *flexible MH* framework of Georgiadis et al. (2024), which we here extend to dynamic contracts – in the context of debt and risk-sharing contracts with LE constraints.

These two MH frameworks have a fundamental difference and a fundamental common property. In the flexible MH framework, the agent (i.e. the borrower in our economies) *chooses a distribution of shocks* in a given compact set. In contrast, in the canonical MH framework the agent *chooses an effort to improve a given distribution of shocks*. Both choices have associated costs with the property that higher costs result in better distributions, in terms of first-order stochastic dominance (FOSD). The same objective – a desired distribution (or FOSD)– requires different IC constraints: in the canonical MH, the IC ensures that the marginal cost of exerting effort equates the associated expected marginal utility gain; while, in the flexible MH, the IC ensures that the marginal cost of choosing a distribution equates the associated expected marginal utility gain.[1]

Associated with the fundamental difference, these two MH frameworks have distinct provisions of incentives. In particular, the canonical approach applies the general contracting enforcement principle of 'the carrot and the stick'. In a principal-agent relationship, the provision of incentives rewards the agents when the outcome is good and punishes it otherwise, which in general means that the *ex-post* value of the contract

[1]The difference can be illustrated with the multi-armed bandit (MAB) problem: in the canonical MAB all the arms have the same range of possible prizes and the agent chooses how much effort to exercise in each one, while in the flexible MAB different arms can have different prizes and the agent chooses (possibly randomly) which arm (or arms) to play.

varies accordingly. For instance, the value of the contract decreases for bad outcomes that may occur due to bad luck rather than a lack of effort. When the contract is a risk-sharing agreement between a risk-averse agent and a risk-neutral principal, the IC constraint disrupts the full risk-sharing that could be achieved with observable effort. The risk distribution improves – in stochastic dominance – with effort but cannot be reduced, since all distributions have the same support.[2]

In opposition, the flexible MH approach applies the contracting enforcement principle of 'reward the cost beyond the minimum performance' rather than tying incentives to realized outcomes, since outcomes *per se* provide no information when the agent chooses distributions. In fact, the agent may choose to reduce risk if this is not too costly. In other words, the enforcement principle of the canonical MH is useless in the flexible MH, and vice versa, since with a given range of outcomes the minimum performance (observable outcome) is independent of effort. In sum, the flexible and the canonical MH are neither substitutes nor a special case of one another.

The principal designs a menu of contracts, each offering specific expected utility to the agent. Since realized outcomes are uninformative, the contracts are not contingent on them. The agent chooses among the menu of contracts offered by the principal. This is incentive compatible as the principal knows the full set of distributions the agent could implement, and designs the contract so that only the desired distribution is incentive-compatible. If the agent picks another distribution (say, a cheaper one) than the one specified in the contract, the Fund's transfer is such that her expected utility would be lower.

In a dynamic context the distinction has long-run opposite effects. As shown by Atkeson and Lucas (1992), in the canonical framework the disruption of the IC constraint increases as a submartingale. That is, with an unbounded concave utility the *ex-post* value of the agent decays as a supermartingale to *immiseration*. In contrast, in the flexible framework, the absence of punishment leads to contracts that avoid the aforementioned immiseration effect. In particular, we show that, as long as the agent's LE constraint is not binding and the agent is not too impatient relative to the principal, the *ex-post* value of the agent increases as a submartingale. The contract therefore features *bliss* as opposed to *immiseration*.

Regarding IC constraints, differentiability allows in the canonical case the First-Order Approach of Rogerson (1985) – extended to our dynamical context in Ábrahám et al. (2025). In the flexible case, the main assumption is that the cost related to the choice of a distribution is Gateau differentiable, resulting in a distributional version of

[2]Otherwise, the principal could elucidate the effort and apply a more severe punishment and reward.

the First-Order Approach that we extend to dynamic contracts. In sum, the extension and characterization of the flexible MH approach of Georgiadis et al. (2024) to dynamic contracts is our first contribution.

We study economies where an impatient and risk-averse sovereign can benefit from borrowing and insuring risks. Risk has an exogenous and an endogenous component, only the latter can be reduced with flexible MH, therefore in this framework risk-sharing is always valuable. We focus on economies where the sovereign borrower has access to private capital debt markets and to a Fund providing debt and insurance with long-term state-contingent Fund contracts, based on a risk-assessment of the borrower. The Fund contract also accounts for two LE constraints: no-default (of the borrower) and no-expected losses (for the private lenders and Fund), in all periods and states. The borrower's default option means to be in default in a Incomplete Market economy with Defaultable (IMD) debt as the only instrument to smooth consumption upon market re-entry.

In the IMD economy, there is the same MH framework as in the economy with the Fund. In particular with flexible MH, the borrower, conditional on an exogenous state, chooses a distribution equating its marginal cost with its expected marginal benefit. The risk-averse borrower has no incentive to relocate the probability mass across different (endogenous) shock levels. Consequently, as we show, in both economies there is a unique choice of a Dirac distribution. In other words, there is *risk reduction* in both economies, although risk-sharing is still valuable since exogenous risk remains. The differences between the two economies being that in the IMD economy there is no risk-sharing and the marginal benefit is determined by the (yelfish) borrower's value function. In contrast, in the Fund contract, there is risk-sharing and the choice itself is a IC constraint, where the corresponding borrower's value function also accounts for the externality effect that the choice has on the risk-sharing contract and on lendery' gains.

To bridge the gap between the canonical and the flexible MH. First, we back-load incentives in the canonical approach based on the following logic. If the LE constraints are not binding in the flexible MH, IC constraints *never* distort risk-sharing. In opposition, in the canonical MH, IC constraints *always* distort risk sharing. We therefore consider long-term canonical MH contracts as a sequence of subprograms. Within subprograms full-risk sharing is preserved, but when one of the LE constraints binds the subprogram terminates and a new subprogram starts with the initial condition accounting for the performance of the previous sub-contract. That is, in the subprogram, IC punishments and rewards are *back-loaded* to the start of the following subprogram. Furthermore, the end of the subprogram is endogenously determined by the binding LE constraints and the punishment-reward mechanism must satisfy the borrower's constraint; that is, it cannot punish when the no-default constraint is binding. This brings the back-loaded design closer to the flexible MH design. Interestingly, it also brings it closer to existing *official lending programs*, where it is common that the end of a relatively short term program is followed by another program, with a new risk-assessment (i.e. based on the previous performance), since official lenders – in particular, in a union of sovereign countries – have a long-term relationship with the borrowing countries.

Regarding flexible MH, we restrain the choice of distributions. In other words, the agent is restricted to choose among a specific family of distributions, each with different costs as before but without the possibility to reduce risk. While in the unrestricted flexible MH, when risk-reduction is not too costly, the optimal distribution is a Dirac distribution, it becomes the closest available approximation of a Dirac in the restricted case. This brings the contract closer to the canonical MH design. Bridging the two distinct frameworks is our third contribution.

After deriving and characterizing the different Fund contracts, we offer a quantitative exploration using benchmark calibrations – closed to Ábrahám et al. (2025) – for the Euro Area stressed economies in the euro crisis (Greece, Italy, Portugal and Spain). Comparing the economies with the Fund: the *Restricted Flexible*, the *Back-Loaded* and the *Canonical* MHs are fairly close to each other, with this enumeration being the borrower's ranking, while the *Flexible* MH Pareto dominates all of them. In particular, the Fund under flexible MH enables the smoothest consumption path for the borrower. The quantitative exploration of the different MH frameworks – calibrating them to the Euro Area stressed countries – is our fourth contribution.

There is a fifth contribution located in our quantitative analysis: *the introduction of flexible MH* to model sovereign debt risk *is literally counterfactual*. We have a reasonable benchmark calibration with the canonical MH in an economy where part of the risk is endogenous. However, in this benchmark economy with flexible MH the borrower sustains a level of debt corresponding to more than three times the level of GDP without ever defaulting and other statistics are also at odds with the historical series. In other words, an alternative calibration with flexible MH as a benchmark does not pass the 'reasonability test'. Nevertheless, we do not conclude from this that flexible MH should not be part of the sovereign debt modeling toolbox, since sovereign debt risk prevention is about choosing (part of) distributions for specific risks (health, climate, etc.).

The paper is organized as follows. Section 1.1 reviews the literature. Section 2 exposes the environment. Sections 3 and 4 develop the Fund contracts under flexible and canonical MH, respectively. Section 5 exposes the back-loaded and restricted flexible Fund contracts. Section 6 contains the quantitative analysis. Section 7 concludes. The Appendix contains the proofs and the details on the data used for the calibration.

1.1 Literature Review

The paper derives optimal contracts between a lender and a borrower and therefore relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994) who considered the case of limited enforcement. The difference with our approach is that we consider two-sided limited enforcement, while the literature has focused on one-sided limited enforcement. We solve the optimal contract by means of the Lagrangian approach of Marcet and Marimon (2019) which has been widely used to account for limited enforcements (e.g. Kehoe and Perri (2002) and Ferrari et al. (2024)) and its combination with moral hazard (e.g. Simpson-Bell (2020) and Ábrahám et al. (2025)). In doing so we describe and contrast the dynamic of the relative Pareto weight under different provisions of incentives.

We develop an optimal contract combining limited enforcement and moral hazard constraints. Our analysis is close to Atkeson (1991) who – similar to Thomas and Worrall (1994) – studies lending contracts in international contexts. However, Atkeson (1991) models moral hazard with respect to consuming or investing the borrowed funds, while we focus on risk management policies. Quadrini (2004) also combines moral hazard and limited enforcement to study when and how contracts are renegotiation-proof. Similarly, **?** shows that the combination of moral hazard and limited enforcement can generate a region of *ex post* inefficiency. This is not the focus of our analysis as our contract is both *ex ante* and *ex post* efficient. In addition, Müller et al. (2019) study dynamic sovereign lending contracts with moral hazard, with respect to reform policy efforts, and limited enforcement. Their characterization of the constrained-efficient allocation is more stylised (normal times are an absorbing state) and focuses on one form of moral hazard only.

Our research contributes to the literature on moral hazard within dynamic macroeconomic models. Building upon the seminal work of Prescott and Townsend (1984), which demonstrated a constrained efficient allocation can be the allocation of competitive equilibrium if the space of contracts satisfy the corresponding incentive compatibility constraints, we extend the flexible moral hazard approach introduced by Georgiadis et al. (2024) to a dynamic framework. We then compare our model's incentive structures with those in the canonical dynamic moral hazard model proposed by Atkeson and Lucas (1992).

In the canonical model, moral hazard results in immiseration due to an incentive-compatible mechanism that rewards high types with greater future utility while penalizing low types with lesser future utility. This mechanism also impedes risk sharing because of the reduced future utility for low types. In contrast, our flexible moral hazard approach leads to what we term "blisy", the antithesis of immiseration, and does not disrupt risk sharing, as incentives are not contingent on realized outcomes. Furthermore, we propose two ways to minimize the disruption to risk sharing in the canonical model. The first one back-loads incentives to offer spans of consumption smoothing, while the second limits the flexibility in the borrower's choice.

Our work more closely contributes to the recent literature on the design of an optimal stability Fund. Roch and Uhlig (2018), Liu et al. (2020) and Callegari et al. (2023) focus on the lender's side of the contract and therefore disregard moral hazard issues. In opposition, Dovis and Kirpalani (2023) account for moral hazard and show that the provision of effort is back-loaded. We build on Ábrahám et al. (2025), where defaultable sovereign debt is transformed into a safe Fund contract, which accounts for moral hazard. They assume that the Fund has an exclusivity contract unlike Liu et al. (2020) and Callegari et al. (2023) who model a Fund which absorbs a minimal amount of debt. We exploit the fact that incentive compatibility constraints are disruptions to perfect risk-sharing. Our contribution is twofold. First, we provide a more comprehensive analysis of moral hazard in the Fund design, describing and contrasting different provisions of incentives and their interactions with limited enforcement constraints. Second, we offer a quantitative exploration using a benchmark calibration for the Euro Area stressed countries.

2 Environment

We introduce flexible moral hazard (MH) in the environment studied in Ábrahám et al. (2025). Consider an infinite-horizon small open economy with a single homogenous consumption good in discrete time. A benevolent government acts as a representative agent and takes decisions on behalf of the small open economy.

In each period, the government receives a stochastic endowment $y \in Y = [\underline{y}, \overline{y}]$ which is drawn from a probability distribution π. The government is able to generate any distribution over Y. We denote by $\mathcal{M}$ the set of Borel probability measures on Y and by $\delta_{y'}$ the Dirac measure generating y' with probability one. Note already that we extend the analysis to a production economy with endogenous labor in Section 6.

The government discounts the future at the rate β, satisfying $\beta \leq 1/(1+r)$, where r is the risk-free world interest rate. The fact that the government is less patient than the lenders implies that it would like to front-load consumption. The government's utility can be defined by $U : \mathbb{R}^+ \times \mathcal{M} \to \mathbb{R}$ and is additively separable. So, if the government chooses a distribution π and consumes c then its instantaneous payoff is $U(c, \pi) \equiv u(c) - v(\pi)$. We make standard assumptions on preferences of consumption. For the distribution choice, we assume that the cost of effort is continuous, strictly convex, Gateaux thrice differentiable and monotone in first-order stochastic dominance. We also normalize the first Gateaux derivative to be zero at $\underline{y}$.[3]

Assumption 1 (Monotonicity, Differentiability and Convexity)**.** *The utility functions from consumption, $u : \mathbb{R}^+ \to \mathbb{R}$, is continuous, strictly increasing and strictly concave. The utility function from effort, $v : \mathcal{M} \to \mathbb{R}$, is continuous, strictly convex, Gateaux thrice differentiable where $v_\pi : K \to \mathbb{R}^+$, $w_\pi : K^2 \to \mathbb{R}^+$ and $z_\pi : K^3 \to \mathbb{R}^+$ denote the first, second and third Gateaux derivative, respectively. Moreover, if the distribution π first-order stochastically dominates $\tilde{\pi}$ then $v(\pi) \geq v(\tilde{\pi})$. Finally, $v_\pi(\underline{y}) = 0$.*

Formally, a Gateaux derivative is defined as follows. The cost function v is Gateaux differentiable at $\pi \in \mathcal{M}$ if there exists a continuous function v_π such that for all $\pi' \in \mathcal{M}$,

$$\lim_{\epsilon \downarrow 0} \frac{v(\pi + \epsilon(\pi' - \pi)) - v(\pi)}{\epsilon} = \int v_\pi(y)(\pi' - \pi)(dy). \tag{1}$$

To define the second and third derivatives it suffices to replace $v(\pi + \epsilon(\pi' - \pi))$ on the left-hand side with $v_{\pi+\epsilon(\pi'-\pi)}(\cdot)$ and $w_{\pi+\epsilon(\pi'-\pi)}(\cdot)$, respectively, and to replace $v_\pi(y)$ on the right-hand side with $w_\pi(\cdot, y)$ and $z_\pi(\cdot, \cdot, y)$, respectively.

To illustrate the shape of such derivatives, we give an example of the cost function that we later use in the quantitative section. Let $L : \mathbb{R} \to \mathbb{R}$ be an increasing, strictly convex and differentiable function and $v(\pi) = L[\int(y - \underline{y})\pi(\mathrm{d}y)]$. The Gateau derivatives are then given by $v_\pi(y) = L'[\int(y - \underline{y})\pi(\mathrm{d}y)](y - \underline{y})$, $w_\pi(i, y) = L''[\int(y - \underline{y})\pi(\mathrm{d}y)](i - \underline{y})(y - \underline{y})$ and $z_\pi(j, i, y) = L'''[\int(y - \underline{y})\pi(\mathrm{d}y)](i - \underline{y})(j - \underline{y})(y - \underline{y})$.

3 The Fund under Flexible Moral Hazard

The *Fund contract* establishes a long-term relationship between the borrower and the Fund by defining a state-contingent sequence of consumption and shock distribution

[3] This is without loss of generality. If v_π is a derivative of v, then $v_\pi + k$ for $k \in \mathbb{R}$ is also a derivative of v.

that maximises the life-time utility of both contracting parties given some initial conditions. It seeks to provide risk-sharing between the borrower and the Fund to the extent possible. However, LE and MH frictions preclude perfect risk-sharing.

The optimal contract is self-enforcing through the presence of two LE constraints. First, we assume that if the borrower ever defaults on the Fund contract, it will not be able to sign a new contract with the Fund and will enter autarky permanently. The Fund contract, however, makes sure that the borrower never finds it optimal to renege the contract. Second, the contract also prevents the Fund from ever incurring undesired expected losses, i.e. undesired permanent transfers.

In addition, the contract also has an incentive compatibility constraint, since the distribution π is non-contractible (i.e. it is private information, or a sovereign right of the borrower). Thus, the long term contract must provide sufficient incentives for the borrower to implement a constrained efficient distribution.

3.1 The Constraints

Given the LE and MH frictions, the Fund has to account for three different constraints. The first one is the LE constraint of the borrower. For any $y^t, t \geq 0$, it should be that

$$\mathbb{E}_t \left[\sum_{j=t}^{\infty} \beta^{j-t} U(c(y^j), \pi_{j+1}) \right] \geq V^D(y_t). \tag{2}$$

The notation is implicit about the fact that expectations are conditional on the implemented distributions of $\{y^j\}_{j=t}^{\infty}$. The borrower's outside option is given by

$$V^D(y_t) = \max_{\pi_{t+1}} \left\{ U(y_t - \phi(y_t), \pi_{t+1}) + \beta \int V^D(y^{t+1}) \pi_{t+1}(\mathrm{d}y^{t+1}) \right\},$$

where $\phi : Y \to \mathbb{R}^+$ is a default penalty with $\phi'(\cdot) \in [0, 1]$. The second constraint is the LE constraint of the Fund. For any $y^t, t \geq 0$, it should hold that

$$\mathbb{E}_t \left[\sum_{j=t}^{\infty} \left(\frac{1}{1+r} \right)^{j-t} \left(y_j - c(y^j) \right) \right] \geq Z(y_t). \tag{3}$$

The finite outside option of the Fund $Z(y_t) \leq 0$ measures the extent of *ex-post* redistribution the Fund is willing to tolerate. That is, if $Z(y_t) < 0$ the Fund is allowed to make a permanent loss in terms of lifetime expected net present value – i.e. the Fund can find better investment opportunities in the international financial market and if it does not renege it is because it has committed to sustaining $Z(y_t) < 0$. Clearly, the level of $Z(y_t)$ has an important impact on the amount of risk sharing in our environment and it can thus be interpreted as the extent of solidarity the Fund is willing to accept in state y_t, as in Tirole (2015).

Finally, the last constraint is the *incentive compatibility* (IC) constraint. Define $V^b(y^t) = \mathbb{E}_t[\sum_{j=0}^{\infty} \beta^j U(c(y^{t+j}), \pi_{t+j+1})]$ as the value of the borrower at time t. For any $y^t, t \geq 0$ and a given consumption schedules $\{c(y^t)\}_{t=0}^{\infty}$, the optimal vector of distributions from the borrower's perspective is

$$\begin{aligned}\pi_{t+1} &= \underset{\tilde{\pi}}{\operatorname{argmax}} \left\{ -v(\tilde{\pi}) + \beta \int V^b(y^{t+1}) \tilde{\pi}(\mathrm{d}y^{t+1}) \right\} \\ &= \underset{\tilde{\pi}}{\operatorname{argmax}} \left\{ \int \left[\beta V^b(y^{t+1}) - v_{\pi_{t+1}}(y^{t+1}) \right] \tilde{\pi}(\mathrm{d}y^{t+1}) \right\},\end{aligned}$$

where the second equality comes from the Gateau differentiability of the cost function $v(\cdot)$ in Assumption 1. In particular, carefully observe the difference between the optimal distribution π_{t+1} and the operand $\tilde{\pi}$ within the integrals. We can then re-scale the maximization problem by stating the gain and cost of effort in relative terms to the *no-effort option*,

$$\pi_{t+1} = \underset{\tilde{\pi}}{\operatorname{argmax}} \left\{ \int \left[\beta \left(V^b(y^{t+1}) - V^b(\underline{y}) \right) - \left(v_{\pi_{t+1}}(y^{t+1}) - v_{\pi_{t+1}}(\underline{y}) \right) \right] \tilde{\pi}(\mathrm{d}y^{t+1}) \right\}.$$

This rescaling is possible as $\beta V^b(\underline{y}) - v_{\pi_{t+1}}(\underline{y})$ is a constant. Since $v_{\pi_{t+1}}(\underline{y}) = 0$ by Assumption 1, the IC constraint is for any $y^{t+1}, t \geq 0$,

$$v_{\pi_{t+1}}(y^{t+1}) = \beta \left(V^b(y^{t+1}) - V^b(\underline{y}) \right). \tag{4}$$

Here, $v_{\pi_{t+1}}(y^{t+1})$ is the Gateaux derivative of the borrower's cost evaluated at the distribution π_{t+1} in the direction of placing *more probability mass* on the realization y^{t+1}. Using the Dirac measure $\delta_{y^{t+1}}$, this direction is represented by the difference $\delta_{y^{t+1}} - \pi_{t+1}$, which captures a shift in probability mass away from the current distribution and toward the specific outcome y^{t+1}. Setting $\pi = \pi_{t+1}$ and $\pi' = \delta_{y^{t+1}}$ in (1), one gets that

$$v_{\pi_{t+1}}(y^{t+1}) = \lim_{\epsilon \downarrow 0} \frac{v(\pi_{t+1} + \epsilon(\delta_{y^{t+1}} - \pi_{t+1})) - v(\pi_{t+1})}{\epsilon} + \int v_{\pi_{t+1}}(y^{t+1}) \pi_{t+1}(\mathrm{d}y^{t+1}).$$

In words, this derivative measures the marginal cost of relocating an infinitesimal amount of probability mass from the current distribution π_{t+1} toward the realization y^{t+1}.[4] Intuitively, it captures how costly it is for the borrower to distort the distribution slightly in favor of y^{t+1}.

[4]This means that global perturbations of the measure (such as $\pi'_{t+1} - \pi_{t+1}$) can be viewed as weighted combinations of local deviations that shift probability mass toward specific realizations (represented by Dirac measures). In that logic $v_{\pi_{t+1}}$ serves as an influence function: its integral against a perturbation direction quantifies the resulting change in cost.

In the context of equation (4), the IC constraint guarantees that the borrower has no incentive to deviate from the distribution π_{t+1} by reallocating probability mass. That is, the marginal cost of increasing the likelihood of any particular endowment realization must equal the marginal gain in continuation value, relative to the baseline endowment $\underline{y}$.

Importantly, in this setup, the borrower directly selects a distribution over outcomes, not merely an action that stochastically determines outcomes. This implies that the borrower can arbitrarily distort the relative likelihood of any subset of endowment realizations. Since these realizations are fully manipulable, they carry no informational content. As a result, the provision of incentives cannot rely on realized endowments. Instead, incentives must be provided entirely through compensating the *marginal cost* of assigning probability mass to each outcome in the support Y, as captured by the Gateaux derivative.

Since we defined $v_{\pi_{t+1}}(y^{t+1})$ by means of the perturbed measure $\pi_\epsilon = (1-\epsilon)\pi_{t+1} + \epsilon\delta_{y^{t+1}}$, equation (4) relies on *local* perturbations of the target distribution π_{t+1}. Hence, for the IC constraint to be valid, the borrower needs not have full flexibility in the choice of distributions (nor monotonicity in first-order stochastic dominance). A local flexibility in the sense that the borrower can generate small perturbations of π_{t+1} in arbitrary directions is enough. We define such capacity as follows

Definition 1 (Local Flexibility)**.** *We say that the borrower enjoys local flexibility, when the target distribution $\ddot{\pi} \in \mathcal{N} \subset \mathcal{M}$ is such that for every $\pi \in \mathcal{M}$, there is some $\epsilon > 0$ for which $\ddot{\pi} + \epsilon(\pi - \ddot{\pi}) \in \mathcal{N}$ is feasible.*

In Section 5, we restrict the borrower's choice of distributions in such a way that Definition 1 does not hold anymore. In that case, endowment realizations become informative and condition the provision of incentives.

By defining the IC constraint in this way, we use the first-order approach. That is we replace the agent's full optimization problem with respect to $\tilde{\pi}$ by its necessary and sufficient first-order condition.[5] This method relies on the Gateaux differentiability of the borrower's objective, as in Georgiadis et al. (2024), and generalizes the classic approach of Rogerson (1985), which assumes a lower-dimensional incentive problem.

[5]Convexity of v is not needed for necessity, only for sufficiency.

3.2 The Long Term Contract

In its extensive form, the *Fund contract* specifies that in state $y^t = (y_0, \ldots, y_t)$, the borrower consumes $c(y^t)$ and chooses the distribution π_{t+1}, resulting in a transfer to the Fund of $y_t - c(y^t)$. With two-sided LE and MH constraints, an optimal Fund contract is a solution to the following Fund problem

$$\max_{\{c(y^t),\pi_{t+1}\}} \mathbb{E}_0 \left[\alpha_{b,0} \sum_{t=0}^{\infty} \beta^t U(c(y^t), \pi_{t+1}) + \alpha_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[y_t - c(y^t) \right] \right]$$
$$\text{s.t.} \quad (2), \ (3), \ \text{and} \ (4), \ \forall y^t, t \geq 0.$$

Note that $(\alpha_{b,0}, \alpha_{l,0})$ are the initial Pareto weights, which are key for our interpretation of the Fund contract as a risk-sharing contract. Given (2), (3) and (4), we take the following interiority assumption to ensure the uniform boundedness of the Lagrange multipliers.

Assumption 2 (Interiority)**.** *There is an $\epsilon > 0$, such that, for all $y_0 \in S$ there is a contract $\{\tilde{c}(y^t), \tilde{\pi}_t\}_{t=0}^{\infty}$ satisfying constraints (2) and (3) when, on the right-hand side, $V^D(y_t)$ and $Z(y_t)$ are replaced by $V^D(y_t) + \epsilon$ and $Z(y_t) + \epsilon$, respectively, and similarly, when in (4) $v_{\pi_{t+1}}(y^{t+1})$ is replaced by $v_{\pi_{t+1}}(y^{t+1}) + \epsilon$ and $=$ is replaced by $\leq$.*

For constraints (2) and (3), this assumption requires that, in spite of the LE constraints, there are strictly positive rents to be shared since otherwise there may not be a constrained-efficient risk-sharing contract. The last part of this assumption is satisfied if a distribution exists that generate a marginal benefit above the marginal cost.[6]

Following Marcet and Marimon (2019) and Mele (2011), we can rewrite the Fund

[6]The first part of the assumption can easily be satisfied since there are gains from risk-sharing in a contract between a risk-averse borrower and a risk-neutral Fund as long as there is a sufficiently high penalty for default $\phi(\cdot)$. The second part of the assumption is also easily met under various cost functions $v(\pi)$ if full risk sharing is not the only feasible allocation.

contract problem as a saddle-point Lagrangian problem:

$$
\begin{aligned}
\text{SP} \min_{\{\gamma_b(y^t),\gamma_l(y^t),\xi(y^{t+1})\}} \max_{\{c(y^t),\pi_{t+1}\}} & \Bigg\{ \mathbb{E}_0 \Bigg[\sum_{t=0}^{\infty} \beta^t \Big(\alpha_{b,t}(y^t) U\left(c(y^t), \pi_{t+1}\right) \\
& - \xi(y^{t+1}) \Big(\beta V^b(\underline{y}) + v_{\pi_{t+1}}(y^{t+1}) \Big) + \gamma_b(y^t) \big[U(c(y^t), \pi_{t+1}) - V^D(y_t) \big] \Big) \\
& + \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \Big(\alpha_{l,t+1}(y^t)[y_t - c(y^t)] - \gamma_l(y^t) \big[y_t - c(y^t) - Z(y^t) \big] \Big) \Bigg] \Bigg\} \\
\text{s.t.} \qquad & \alpha_{b,t+1}(y^{t+1}) = \alpha_{b,t}(y^t) + \gamma_b(y^t) + \xi(y^{t+1}), \\
& \alpha_{l,t+1}(y^t) = \alpha_{l,t}(y^t) + \gamma_l(y^t), \\
& \alpha_{b,0}(y^0) \equiv \alpha_{b,0}, \alpha_{l,0}(y^0) \equiv \alpha_{l,0} \text{ given,}
\end{aligned}
$$

where $\gamma_b(y^t)$, $\gamma_l(y^t)$ and $\xi(y^{t+1})$ are the Lagrange multipliers of the LE constraints in (2) and in (3), and the IC constraint in (4), respectively, in state y^{t+1}.

The above formulation of the problem defines two new co-state variables $\alpha_b(y^t)$ and $\alpha_l(y^t)$, which represent the temporary Pareto weights of the borrower and the Fund respectively. These variables are initialized at the original Pareto weights $(\alpha_{b,0}, \alpha_{l,0})$ and become time-variant because of the LE and MH frictions. In particular, a binding LE constraint of the borrower (Fund) will imply a higher co-state variable of the borrower (Fund) so that it does not leave the contract. In addition, the MH friction implies that the borrower's co-state variable will increase as $\xi(y^{t+1}) \geq 0$ under Assumption 2.

Given the homogeneity of degree one of the maximization problem in $(\alpha_{b,t}, \alpha_{l,t})$, only relative Pareto weights, defined as $x_t(y^t) \equiv \alpha_{l,t}(y^t)/\alpha_{b,t}(y^t)$, matter for the allocations, and this allows us to reduce the dimensionality of the co-state vector and write the problem recursively by using a convenient normalization. Let $\eta \equiv \beta(1+r) \leq 1$ and normalize the multipliers as follows

$$
\nu_b(y^t) = \frac{\gamma_b(y^t)}{\alpha_{b,t}(y^t)}, \; \nu_l(y^t) = \frac{\gamma_l(y^t)}{\alpha_{l,t}(y^t)} \text{ and } \varrho(y^{t+1}) = \frac{\xi(y^{t+1})}{\alpha_{b,t}(y^t)}.
$$

The *Saddle-Point Functional Equation* (**SPFE**) — i.e. the saddle-point version of Bellman's equation — is given by

$$
\begin{aligned}
FV(y,x) = \text{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,\tilde{\pi}\}} & \Big\{ x \Big[(1+\nu_b) U(c,\tilde{\pi}) - \nu_b V^D(y) \Big] \qquad (5) \\
& + \Big[(1+\nu_l)[y-c] - \nu_l Z(y) \Big] \\
& + \int \left[\frac{1+\nu_l}{1+r} FV(y', x'(y')) - x\varrho(y') \Big(v_{\tilde{\pi}}(y') + V^b(\underline{y}, x'(\underline{y})) \Big) \right] \tilde{\pi}(\mathrm{d}y') \Big\}
\end{aligned}
$$

$$
\text{s.t.} \quad x'(y') \equiv \overline{x}'(y) + \hat{x}'(y') = \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varrho(y')}{1+\nu_l} \right] \eta x, \qquad (6)
$$

Equation (23) gives the law of motion of the relative Pareto weight in recursive form. The prospective weight $x'(y')$ can be separated into two parts: the update due to the borrower's LE constraint $\overline{x}'(y)$ and the update due to the IC constraint $\hat{x}'(y')$. Furthermore, the Fund's value functions can be decomposed as follows

$$FV(y,x) = xV^b(y,x) + V^l(y,x) \text{ with} \tag{7}$$

$$V^l(y,x) = y - c + \frac{1}{1+r}\int V^l(y', x'(y'))\pi(\mathrm{d}y'), \tag{8}$$

$$V^b(y,x) = U(c,\pi) + \beta\int V^b(y', x'(y'))\pi(\mathrm{d}y'). \tag{9}$$

The policy functions for consumption and labor of the Fund contract must solve the first-order conditions of the SPFE. In particular, $c(y,x)$ satisfies

$$u'(c(y,x)) = \frac{1+\nu_l(y,x)}{1+\nu_b(y,x)}\frac{1}{x} \tag{10}$$

This conditions is standard as the borrower's consumption is determined by its endogenous relative Pareto weight.

Regarding the optimal distribution, define $\widetilde{FV}(y', x'(y')) \equiv x'(y')\left[V^b(y', x'(y')) - V^b(\underline{y}, x'(\underline{y}))\right] + \left[V^l(y', x'(y')) - V^l(\underline{y}, x'(\underline{y}))\right]$ which corresponds to $FV(y', x'(y'))$ except for the addition of $V^l(\underline{y}, x'(\underline{y}))$. Since this additional term is constant given $\{\nu_b, \nu_l, \varrho(y')\}$ and $\{c\}$, $\tilde{\pi}$ maximizes the Fund's objective function if it maximizes

$$\int \left[\frac{1+\nu_l}{1+r}\widetilde{FV}(y', x'(y')) - x(1+\nu_b)v_\pi(y') - x\varrho(y')v_{\tilde{\pi}}(y')\right]\tilde{\pi}(\mathrm{d}y').$$

As a result, the optimal distribution from the Fund's perspective solves

$$\begin{aligned}\Lambda_\pi(y') \equiv & x\left(1+\nu_b+\varrho(y')\right)\left[\beta\left(V^b(y', x'(y')) - V^b(\underline{y}, x'(\underline{y}))\right) - v_{\pi(y')}(y')\right] \\ & + \frac{1+\nu_l}{1+r}\left[V^l(y', x'(y')) - V^l(\underline{y}, x'(\underline{y}))\right] - x\varrho(y')\int w_\pi(i,y')\pi(\mathrm{d}i) = 0.\end{aligned}$$

The first line of the expression is simply (4) meaning that the Fund contract has to compensate the borrower for choosing a certain level of effort. The second line of the expression accounts for the effect on the Fund itself plus the marginal relaxation/tightening effect when there is a change in the probability of shock y'.[7] As (4) holds, the optimal distribution is such that

$$\frac{1}{1+r}\left[V^l(y', x'(y')) - V^l(\underline{y}, x'(\underline{y}))\right] = x\frac{\varrho(y')}{1+\nu_l}\int w_\pi(i,y')\pi(\mathrm{d}i). \tag{11}$$

For completeness of the argument, we provide a definition of the Fund contract and subsequently show existence and uniqueness extending the proof of Marcet and Marimon (2019) and Ábrahám et al. (2025) to our environment.

[7]More precisely, the second Gateau derivative of $v(\cdot)$, $w_\pi(i,y')$, represents the change in the marginal cost of generating the shock i associated with a slight increase in the probability of y'.

Definition 2 (Fund Contract). *Given an initial relative Pareto weights $x_0(y_0)$ and outside options $\{V^D(y), Z(y)\}$, the policies for the allocations $\{c(y,x), \pi(y,x)\}$, multipliers $\{\nu_b(y,x), \nu_l(y,x), \varrho(y,x,y')\}$, value functions $\{V^b(y,x), V^l(y,x)\}$, relative Pareto weight $\{x'(y,x,y')\}$ are a recursive constrained-efficient Fund contract if they satisfy conditions (23)-(11) for all (y,x).*

Proposition 1 (Existence and Uniqueness). *Given Assumptions 1 and 2, for any y_0, $x(y_0)$, and outside options $\{V^D(y), Z(y)\}$, there is a unique recursive constrained-efficient Fund contract.*

Following Ábrahám et al. (2025), we use the term recursive constrained-efficient Fund contract because it is optimal, given the constraints imposed on it, and it has a recursive structure. In the rest of the paper we simply refer to the *Fund contract.* This contract serves as the policy instrument of the Fund. In its design, it considers the constraints of the borrower and the Fund and determines the appropriate policies on labor and consumption. Regarding the distribution decision, the Fund functions as a Principal in a Principal-Agent framework, taking the borrower's first-order condition as a given. We next characterize the optimal distribution and how this interacts with the LE constraint.

3.3 Moral Hazard and Limited Enforcement

We first characterize the optimal distribution before establishing the long run properties of the contract. Following Georgiadis et al. (2024), we can re-formulate (11) using the same approach as we did to derive the IC constraint. In particular, the optimal distribution from the Fund's perspective solves

$$\pi = \underset{\tilde{\pi}}{\operatorname{argmax}} \int \Lambda_\pi(y')\tilde{\pi}(\mathrm{d}y').$$

Observe again the distinction between the optimal distribution π and the operand $\tilde{\pi}$. Given this, maximizing $\int \Lambda_\pi(y')\tilde{\pi}(\mathrm{d}y')$ over all probability distributions is equivalent to concentrating the probability mass on the set of maximizers of $\Lambda_\pi(y')$. That is, the expectation is maximized when $\tilde{\pi}$ assigns all mass to the $y' \in Y$ that maximize $\Lambda_\pi(y')$. Therefore

$$\operatorname{supp} \pi \subseteq \underset{y' \in Y}{\operatorname{argmax}} \ \Lambda_\pi(y').$$

This means that whenever $\Lambda_\pi(y')$ is strictly concave, there is only one y' that maximizes $\Lambda_\pi(y')$. The problem of choosing an optimal distribution is therefore the same as the one of choosing y' directly.[8] The following proposition formalizes this argument.

[8]We work under strict concavity. However, it is enough to have strict quasiconcavity.

Proposition 2 (Optimal Distribution). *If $w_\pi(\cdot, y')$ is strictly convex in y' for every π, then $\Lambda_\pi(y')$ is strictly concave for every π and the optimal distribution has at most one y' in its support. Given the Dirac measure $\delta_{y'}$, the Fund's problem therefore reduces to*

$$FV(y,x) = \text{SP} \min_{\{\nu_b,\nu_l,\varrho(y')\}} \max_{\{c,y'\}} \Bigg\{ x\Big[(1+\nu_b)U(c,\delta_{y'}) - \nu_b V^D(y)\Big] + \Big[(1+\nu_l)[y-c] - \nu_l Z(y)\Big] + \frac{1+\nu_l}{1+r} FV(y', x'(y')) - x\varrho(y')\Big(v_{\delta_{y'}}(y') + V^b(\underline{y}, x'(\underline{y}))\Big)\Bigg\}.$$

The proposition is made of two parts. First, the strict convexity of the second Gateau derivative of $v(\pi)$ implies that Λ_π is strictly concave since the Fund's value is concave in y. Second, when Λ_π is strictly concave, the optimal distribution choice collapses to a Dirac distribution. More generally, this means that the borrower's flexibility in the choice of distribution enables a complete reduction of risk. In other words, the borrower not only can but also find it optimal to eliminate any stochasticity in y'. In Section 6, we introduce an exogenous shock preventing complete risk reduction.

The following lemma provides a characterization of the interaction between LE and MH constraints.

Lemma 1. *When $y' = \underline{y}$, then $\varrho_\pi(y') = 0$. Otherwise, $\varrho_\pi(y') > 0$.*

The lemma states that when the borrower chooses the lowest possible y', the multiplier attached to the IC constraint is zero. Otherwise, it is strictly positive. The rationale behind this result is the following. The borrower incurs zero cost when choosing $\underline{y}$. However, for any $y' > \underline{y}$, the cost is positive and the borrower needs to be compensated accordingly. Hence, when $y' = \underline{y}$, the borrower only gets the basis value $V^b(\underline{y})$ meaning that $\varrho_\pi(y') = 0$. For any other realization, $\varrho_\pi(y') > 0$ to compensate the borrower for incurring more costs.

A direct corollary is that the law of motion of the relative Pareto weight is a left bounded positive submartingale. To see this, take the expectations of the law of motion

$$\mathbb{E}x'(y') \equiv \mathbb{E}_t\left[\bar{x}'(y) + \hat{x}'(y')\right] = \mathbb{E}\left[\frac{1+\nu_b(y)}{1+\nu_l(y)}x(y) + \frac{\varrho(y')}{1+\nu_l(y)}x(y)\right]\eta,$$

where $\hat{x}'(y')$ accounts for the dynamic effect of the MH constraint. There are two forces working against the multiplier on the IC constraint $\varrho(y')$: impatience $\eta \leq 1$ and the Fund's LE constraint $\nu_l(y) \geq 0$. Hence, with neither the Fund's LE constraint nor impatience, we get that $\mathbb{E}x'(y') \geq x(y)$. This means that without either one of these two elements, the relative Pareto weight would go towards infinity. This is the reverse of the *immiseration* result of Atkeson and Lucas (1992). We call it *bliss*.

Corollary 1 (Bliss)**.** *When $\eta = 1$ and $\nu_l(y) = 0$ in all states, $\mathbb{E}x'(y') \geq x(y)$.*[9]

This result is important as it uncovers a different provision of incentives than the canonical MH. The Fund does not rely on the realization of y' to provide incentives but compensates the borrower for the cost it incurred. Such mechanism is more risk sharing friendly as there is no punishment for low realization of y'. In the next section we analyze such problem and contrast it with the outcome of Lemma 1.

4 The Fund under Canonical Moral Hazard

We switch to the canonical MH. So far, the borrower could choose *any* distribution π directly with increasing cost in first-order stochastic dominance. In what follows, it losses its capacity to manipulate in an arbitrary way the relative likelihood of any collection of y'. More precisely, the borrower chooses effort e, which translates into first-order stochastic dominance over *given* distributions. Effort is not contractible and affects the distribution globally – as opposed to locally. As one will see, the canonical and the flexible MH have distinct provisions of incentives and neither of them is a special case of the other.

4.1 The Constraints

The borrower's choice of distribution is restricted to a subset of $\mathcal{Q} \subset \mathcal{M}$. In particular, define Q as a mixture distribution $Q = \varpi(e)Q_L + (1 - \varpi(e))Q_H$ for $Q_L, Q_H \in \mathcal{Q}$ with a weighting function $\varpi : [0, 1] \to [0, 1]$. The borrower can manipulate the weights by choosing the (non-contractible) effort $e \in [0, 1]$ to change Q. We assume that Q_H first-order stochastically dominates Q_L and that $\varpi(e)$ is continuous, strictly decreasing and concave. As it is clear from Definition 1, the borrower does not anymore enjoy local flexibility. This is because any change in e has global effects on the distribution.

To recover the formulation of the canonical MH problem, we consider a different effort cost than what we had so far. The cost of effort $\hat{v} : [0, 1] \to \mathbb{R}$ is a mapping from $[0, 1]$ instead of $\mathcal{M}$. We then have that $\hat{U}(c, e) = u(c) - \hat{v}(e)$.

Notice that the structure of the cost function $\hat{v}(e)$ prevents us to use the argument based on the Gateau differentiability. In the next section we analyze the case of a restricted choice of distributions with the previously adopted cost function $v(\pi)$.

[9]Note that a sufficient condition for $\nu_l(y) = 0$ is that $Z(y)$ is negative enough for all y.

Given the re-definition of the instantaneous utility function – which is now defined over e instead of π, the LE constraint of the borrower reads

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty}\beta^{j-t}\hat{U}(c(y^j),e(y^j))\right]\geq \hat{V}^D(y_t), \tag{12}$$

where the outside option is given by

$$\hat{V}^D(y_t)=\max_e\left\{\hat{U}(y_t-\phi(y_t),e)+\beta\int \hat{V}^D(y^{t+1})Q(y^{t+1}|y_t,e)(\mathrm{d}y^{t+1})\right\}.$$

The LE constraint of the Fund is unchanged compared to (3). We nevertheless repeat it below. For any $y^t, t\geq 0$, it should hold that

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t}(y_j-c(y^j))\right]\geq Z(y_t). \tag{13}$$

The main change relates to the IC constraint. Instead of choosing an entire distribution, the borrower picks an effort level. Define $\hat{V}^b(y^t)=\mathbb{E}_t[\sum_{j=0}^{\infty}\beta^j\hat{U}(c(y^{t+j}),e(y^{t+j}))]$ as the value of the borrower at time t. The optimal choice of effort is given by

$$e(y^t)=\underset{\tilde{e}}{\operatorname{argmax}}\left\{\hat{U}(c(y^t),\tilde{e})+\beta\int \hat{V}^b(y^{t+1})Q(y^{t+1}|y_t,\tilde{e})(\mathrm{d}y^{t+1})\right\}.$$

The IC constraint is therefore

$$\hat{v}_e(e(y^t))=\beta\int \hat{V}^b(y^{t+1})\partial_e Q(y^{t+1}|y_t,e(y^t))(\mathrm{d}y^{t+1}). \tag{14}$$

For the first-order approach of Rogerson (1985) to be valid, the cumulative distribution function of y' should be differentiable, convex and satisfy the monotone likelihood-ratio condition. This mirrors our Assumption 1. However, the similarity with the flexible MH stops here. The IC constraint in (14) relies on the informativeness of the realization of y'. This is because the information content of a specific realization can be directly measured by the relative likelihood given that Q is contractible, whereas e is not. In the case of flexible MH, the probability distribution is a choice variable which is not contractible. This precludes any informativeness of the shock realization.

Assumption 3 (*Differentiability, Monotonicity and Convexity*)**.** *The utility function from effort, $\hat{v}:[0,1]\to\mathbb{R}$, is continuous, convex and twice differentiable. For every y, if $e\geq\tilde{e}>0$ the ratio $\frac{Q(y'|y,\tilde{e})}{Q(y'|y,e)}$ is nonincreasing in y', and, for every (e,y), $Q_p(e,y)=\int_{p\subseteq Y}Q(y'|y,e)(\mathrm{d}y')$ is differentiable in e, with $\partial_e Q_p(e,y)\leq 0$ and $\partial_e^2 Q_p(e,y)\geq 0$.*

Assumption 3 generalizes the assumptions of Rogerson (1985) so that we can apply his first-order condition approach in a simple static Pareto-optimization problem to our dynamic contracting problem with LE and MH frictions.

4.2 The Long Term Contract

With two-sided LE and MH constraints, the optimal Fund contract is a solution to the following maximization problem

$$\max_{\{c(y^t),e(y^t)\}} \mathbb{E}_0\left[\alpha_{b,0}\sum_{t=0}^{\infty}\beta^t\hat{U}(c(y^t),e(y^t)) + \alpha_{l,0}\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t\left[y_t - c(y^t)\right]\right]$$
$$\text{s.t.} \quad (12),\ (13) \text{ and } (14),\ \forall y^t, t \geq 0.$$

In terms of structure, the Fund problem is very similar to what we had under flexible MH. The main change is that the government exercises effort $e(y^t)$ instead of directly choosing a distribution π_{t+1}. As before, to ensure the uniform boundedness of the Lagrange multipliers, we posit an interiority assumption.

Assumption 4 (*Interiority*)**.** *There is an $\epsilon > 0$, such that, for all $y_0 \in Y$ there is a program $\{\tilde{c}(y^t), \tilde{e}(y^t)\}_{t=0}^{\infty}$ satisfying constraints (12) and (13) when, on the right-hand side, $\hat{V}^D(y_t)$ and $Z(y_t)$ are replaced by $\hat{V}^D(y_t) + \epsilon$ and $Z(y_t) + \epsilon$, respectively, and similarly, when in (14) $\hat{v}_e(e(y^t))$ is replaced by $\hat{v}_e(e(y^t)) + \epsilon$ and $=$ is replaced by $\leq$.*

The interiority of effort can be guaranteed if full risk sharing is not the only feasible allocation and appropriate conditions are imposed on the cost $\hat{v}(e)$ and benefit $Q(y'|y,e)$ of effort. Following the previous section, we can formulate the Fund problem in recursive form. We find that the SPFE is given by

$$\hat{FV}(y,x) = \text{SP}\min_{\{\nu_b,\nu_l,\varrho\}}\max_{\{c,e\}}\Big\{x\Big[(1+\nu_b)\hat{U}(c,e) - \nu_b\hat{V}^D(y) - \varrho\hat{v}_e(e)\Big] \tag{15}$$
$$+\left[(1+\nu_l)(y-c) - \nu_l Z(y)\right] + \frac{1+\nu_l}{1+r}\int \hat{FV}(y',x'(y'))Q(y'|y,e)(\mathrm{d}y')\Big\}$$
$$\text{s.t. } x'(y') \equiv \bar{x}'(y) + \hat{x}'(y') = \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varphi(y'|y)}{1+\nu_l}\right]\eta x, \tag{16}$$
$$\varphi(y'|y) = \varrho\frac{\partial_e Q(y'|y,e)}{Q(y'|y,e)}.$$

The Fund's value functions can be decomposed as in the case with flexible MH. Similarly, the policy functions for consumption is the solution to (10). This is because of additive separability in the utility function. Hence, the formulation of the MH does not directly affect the formulation of this first-order condition.

Notice that the multiplier $\varphi(y'|y)$ is defined as $\varrho\frac{\partial_e Q(y'|y,e)}{Q(y'|y,e)}$. It does not explicitly depend on e since, as multiplier, the action is taken as given. Moreover, it can be positive or negative depending on the sign of $\partial_e Q(y'|y,e)$. This reflects the main difference with the flexible MH approach we discussed previously. As the choice of distribution is restricted, the relative likelihood is informative about the realization of y'. In particular, the Fund acting as the principal will punish the borrower acting as the agent when a bad outcome realizes (i.e. $\partial_e Q(y'|y,e) < 0$) and will reward when a good outcome realizes (i.e. $\partial_e Q(y'|y,e) > 0$). Note that bad outcomes may occur here due to bad luck rather than a lack of effort.

The effort policy $e(y,x)$ is determined by the first order condition of the SPFE with respect to e, which can be conveniently expressed as:

$$\begin{aligned}\hat{v}'(e(y,x)) = \beta \int \partial_e Q(y'|y,e)\hat{V}^b(y',x'(y'))(\mathrm{d}y') \\ + \frac{1+\nu_l(y,x)}{1+\nu_b(y,x)}\frac{1}{x}\frac{1}{1+r}\int \partial_e Q(y'|y,e)\hat{V}^l(y',x'(y'))(\mathrm{d}y') \\ - \frac{\varrho(y,x)}{1+\nu_b(y,x)}\left[v''(e(y,x)) - \beta\int \partial_e^2 Q(y'|y,e)\hat{V}^b(x'(y'),y')(\mathrm{d}y')\right].\end{aligned} \tag{17}$$

Equation (17) balances the marginal cost of effort with the benefits. The first line is the life-time utility benefit of effort to the borrower; the second line is the marginal benefit of effort to the Fund; the third line accounts for the marginal relaxation/tightening effect of the MH constraint (14) when there is a change in effort. With contractible effort, the Fund problem would not have the IC constraint (14) and the effort decision would be given by the first two lines, with the second one accounting for the social value of effort. In contrast, with non-contractible effort, as we assume, constraint (14) is present and the first line is equal to zero. In this case, (17) reduces to

$$\begin{aligned}\frac{1}{1+r}\int \partial_e Q(y'|y,e)\hat{V}^l(y',x'(y'))(\mathrm{d}y') \\ = \vartheta(y,x)\left[v''(e(y,x)) - \beta\int \partial_e^2 Q(y'|y,e)\hat{V}^b(y',x'(y'))(\mathrm{d}y')\right],\end{aligned} \tag{18}$$

where $\vartheta(y,x) \equiv \frac{x\varrho(y,x)}{1+\nu_l(y,x)}$ can be interpreted as the marginal value of relaxing the IC constraint in terms of the Fund's valuation; that is, (18) accounts for the external effect of effort on the Fund's value through its effect on the IC constraint. Note that, although the IC constraint implies that only the borrower's returns affect the effort decision directly, the benefits represented in (18) will affect incentives as they affect ϱ and hence the whole future path of allocations through (16).

The definition of the Fund contract can be easily adapted from Definition 2. The interested reader can refer to Ábrahám et al. (2025) who also provide a proof of existence and uniqueness that we do not repeat here.

4.3 Limited Enforcement and Moral Hazard

In the economies we study, with the need of risk-sharing, avoiding default or undesired permanent transfers, MH problems arise when these problems could be alleviated with effort, but such effort is not contractible. Therefore it is reasonable to model contracts with LE constraints that satisfy the following property:

Definition 3. *The LE constraints* (12) *and* (13) *satisfy the 'no-free-lunch condition' if, given* (y, x)*, whenever* $\nu_b(y', x'(y, x, y')) > 0$*, then* $\partial_e Q(y'|y, e) > 0$ *and whenever* $\nu_l(y', x'(y, x, y')) > 0$*, then* $\partial_e Q(y'|y, e) < 0$*, respectively.*

Conversely, if $\partial_e Q(y'|y, e) = 0$ (or the inequality signs were reversed) exercising more effort would not have any effect on the LE constraints (or a perverse effect) and, on those grounds, MH would not be an issue. The following lemma provides a characterization of the interaction between LE and MH constraints.

Lemma 2. *Under Assumption 3, in the Fund contract:*

1. *LE constraints have an effect on the expected law of motion of the Pareto weights, when they are binding; in contrast, MH constraints do not have an effect on* $\{\mathbb{E}x'\}$*, even if they bind; i.e.* $\mathbb{E}x' = \bar{x}'(y)$*.*
2. *If LE constraints satisfy the 'no-free-lunch condition', MH constraint make the borrower's LE constraint* (12) *more likely to bind and the Fund's LE constraint* (3) *less likely to bind and, in both cases,* $\mathbb{E}_t \frac{1}{u'(c')}$ *increases.*

To see the first point, note that, given (16),

$$\int Q(y'|y, e(y))\hat{x}'(y')(\mathrm{d}y') = 0,$$

since independently of effort we have $\int Q(y'|y, e(y))(\mathrm{d}y') = 1$ implying that $\int \partial_e Q(y'|y, e(y))(\mathrm{d}y') = 0$. Therefore $\mathbb{E}\hat{x}' = 0$ and $\mathbb{E}x' = \bar{x}'(y)$. Alternatively, the expected law of motion of x can also be expressed as

$$\mathbb{E}x' = \mathbb{E}\left[\frac{1}{u'(c')}\frac{1 + \nu_l'(y')}{1 + \nu_b'(y')}\right] = \frac{1}{u'(c)}\eta,$$

where the last equality is the *inverse Euler equation* of the recursive contract (Ábrahám et al. (2025), Lemma 4).

To see the second point, note that, since the LE constraint multipliers are either zero or at most one of the two is positive, we have that

$$\mathbb{E}_t \frac{1}{u'(c')} = \mathbb{E}_t \left[x' \frac{1 + \nu_b'(y')}{1 + \nu_l'(y')}\right].$$

If LE constraints satisfy the 'no-free-lunch condition', the borrower's LE constraint is more likely to bind, while the Fund's LE constraint is less likely to bind and, as a result, in both cases expected consumption increases.

5 Flexible vs. Canonical Moral Hazard

In this section, we contrast the two MH formulations. We first back load the incentives in the canonical MH by splitting the Fund contract into a sequence of subcontracts. We subsequently restrict the choice of distributions in the flexible MH framework.

5.1 Back-loaded moral hazard

In the canonical MH problem, the provision of incentives is generated by a system of rewards and punishments associated with the moral-hazard constraint (14). Such system is not unique. We analyze a Fund contract which consists of an infinite sequence of subprograms, whereby within each subprogram rewards and punishment are *back-loaded* to the end.

The length of each subprogram is directly determined by the binding LE constraints. The reason is that whenever a subprogram would violate one of the LE constraints, one of the contracting parties would find it optimal to terminate the contract. Hence, the binding LE constraints endogenously determine the subprogram's end. When this happens, we say that the subprogram resets.

The Fund contract can be expressed as the solution to a sequence of sub-contracts. As a subprogram resets when one of the LE constraint binds, the start of a subprogram is such that

$$
\begin{aligned}
\hat{FV}(y,x) = \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,e\}} \Big\{ & x\left[(1+\nu_b)(u(c)-\hat{v}(e)) - \nu_b \hat{V}^D(y) - \varrho v'(e)\right] \\
& + \left[(1+\nu_l)(y-c) - \nu_l Z(y)\right] \\
& + \frac{1+\nu_l}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(y',x'(y'))\}}\hat{FV}(y',x'(y')) + (1-\mathbb{I}_{\{(y',x'(y'))\}})\overline{FV}(y',x'(y'),\bar{x}') \mid y,e\right] \Big\} \\
\text{s.t.} \quad & x'(y') = \eta x \frac{1+\nu_b+\varphi(y'|y)}{1+\nu_l}, \\
& \bar{x}' = \eta x \frac{1+\nu_b}{1+\nu_l},
\end{aligned}
$$

where $\varphi(y'|y) = \varrho\frac{\partial_e Q(y'|y,e)}{Q(y'|y,e)}$ as in Section 4 and $\mathbb{I}_{\{(y',x'(y'))\}}$ is an indicator function where $\mathbb{I}_{\{(y',x'(y'))\}} = 1$ if one of the LE constraints is binding – i.e. $\nu_b(y',x'(y')) +$

$\nu_l(y', x'(y')) > 0$. In other words, $\mathbb{I}_{\{(y',x'(y'))\}} = 1$ indicates when the subprogram resets. Then within the subprogram

$$
\begin{aligned}
\overline{FV}(y,x,\bar{x}) = \min_{\{\varrho\}} \max_{\{c,e\}} \Big\{ & \bar{x}u(c) - x\big[\hat{v}(e) + \varrho v'(e)\big] + (y-c) \\
& + \frac{1}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(y',x'(y'))\}}\hat{FV}(y',x'(y')) + (1-\mathbb{I}_{\{(y',x'(y'))\}})\overline{FV}(y',x'(y'),\bar{x}') \mid y, e\right] \Big\} \\
\text{s.t.} \quad & x'(y') = \eta x\left[1+\varphi(y'|y)\right], \\
& \bar{x}' = \eta\bar{x}.
\end{aligned}
$$

As it can be seen, within the subprogram $x'(y')$ is the *latent multiplier* which cumulates past incentives. The consumption policy is given by the same first-order condition, (10), resulting in $c(\overline{x}, s)$, while the optimal effort $e(x, s)$ requires a reformulation of (18). For this, it is useful to recall that, as in the benchmark Fund contracts, $\hat{FV}(y, x) = x\hat{V}^b(y, x) + V^l(y, x)$. However, $\overline{FV}(y, x, \overline{x})$ depends on $\overline{x}$ and x; therefore, we first decompose

$$
\begin{aligned}
\overline{V}^b_1(\overline{x}, s) &= u(c(\overline{x}, s)) + \beta\mathbb{E}\left[\overline{V}^b_1(y', x'(y'))\right], \\
\overline{V}^b_2(y, x) &= -\hat{v}(e(x, s)) + \beta\mathbb{E}\left[\overline{V}^b_2(y', x'(y'))\right],
\end{aligned}
$$

then the value of the borrower is simply $\overline{V}^b(y, x, \overline{x}) = \overline{V}^b_1(\overline{x}, s) + \overline{V}^b_2(y, x)$ implying that

$$
\overline{FV}^b(y, x, \overline{x}) = \overline{x}\overline{V}^b_1(\overline{x}, s) + x\overline{V}^b_2(y, x) + \overline{V}^l(y, x, \overline{x}).
$$

Note that, except for the distinction between x and $\overline{x}$, $\overline{FV}(y, x, \overline{x})$ is the same as $\hat{FV}(y, x)$ since we can always incorporate in the minimization $\{\nu_b, \nu_l\}$ which will satisfy $\nu_b = \nu_l = 0$, by construction, within the subprogram. This allows us to express a unique first-order condition for the effort policy.

The presence of subprograms enhances risk sharing compared to the canonical MH. The reason behind this is that, within a subprogram, there is perfect consumption smoothing adjusted for the relative impatience of the borrower. As one can see from the law of motion of $\bar{x}$ together with (10), consumption is entirely deterministic and decays at rate η as long as the subprogram runs. Conversely, when the subprogram resets, consumption is adjusted up if the latent multiplier x is larger than $\overline{x}$. In other words, consumption increases if the borrower accumulated enough good realizations of y' in the past. The opposite is true when $x < \overline{x}$ meaning that the borrower accumulated too few good realizations of y' in the past. However, the end of the subprogram is endogenously determined by the binding LE constraints and the punishment-reward mechanism must satisfy the borrower's constraint; that is, it cannot punish when the borrower's LE constraint is binding. This design makes the Fund contract closer to the design under flexible MH.

It is important to note that the back-loaded structure mirrors the existing framework of sovereign lending programs implemented by international multilateral lenders, such as the IMF. These lenders offer relatively short-term lending programs. These programs are often followed by subsequent arrangements, each contingent on a new risk assessment that takes into account the borrowing country's previous performance. This iterative process ensures that the lending is aligned with the evolving economic conditions and reform progress of the recipient country, thereby aiming to enhance the effectiveness and sustainability of the financial support provided.

5.2 Restricted flexible moral hazard

Besides the back-loading of incentives, there is another way to bridge the gap between the flexible and the canonical MH. In Section 3, the borrower can choose any distribution of π while incurring a cost $v(\pi)$ which is a mapping from $\mathcal{M}$ to the real. We relax the assumption on the distribution choice and consider that the borrower's choice is restricted to a subset of $\mathcal{Q} \subset \mathcal{M}$. As in Section 4, the borrower can choose among a family of distribution $Q(e) = \varpi(e)Q_L + (1 - \varpi(e))Q_H$ by exerting effort $e \in [0, 1]$ which leads to a cost $v(Q(e))$.

Following our argument in Section 3, we can formulate the IC constraint as the outcome of the following maximization problem

$$Q = \operatorname*{argmax}_{\tilde{Q}} \left\{ -v(\tilde{Q}) + \beta \int V^b(y')\tilde{Q}(\mathrm{d}y') \right\}.$$

Given the restriction on the choice of distribution, this maximization problem can be reformulated as a maximization over the level of effort e. More precisely, one can write

$$\begin{aligned} e(y) &= \operatorname*{argmax}_{\tilde{e}} \left\{ -v(Q(\tilde{e})) + \beta \int V^b(y')Q(\tilde{e})(\mathrm{d}y') \right\} \\ &= \operatorname*{argmax}_{\tilde{e}} \left\{ \int \left[\beta V^b(y') - v_{Q(e)}(y') \right] Q(y'|y, \tilde{e})(\mathrm{d}y') \right\}, \end{aligned}$$

where the second equality comes from the Gateau differentiability in Assumption 1. The IC constraint is therefore

$$\int \left[\beta V^b(y') - v_{Q(e)}(y') \right] \partial_e Q(y'|y, e(y))(\mathrm{d}y') = 0. \tag{19}$$

As one can see the expression is similar to the IC constraint (14). In particular, given the presence of $\partial_e Q(y'|y, e(y))$, the provision of incentives relies on the informativeness of the realization of y'. This differs from the flexible MH studied in Section 3. The reason is that the borrower does not enjoy local flexibility. In other words, any change in e has a global effect on the target distribution enabling the Fund to measure the relative likelihood of a specific realization. We therefore conclude that what distinguish the flexible MH and the canonical MH is both the structure of the the cost of effort and the relative flexibility towards which the borrower can allocate likelihood.

Given this, the Fund contract in recursive form reads as follows

$$\begin{aligned}
FV(y,x) = \mathrm{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,e\}} &\Big\{ x\Big[(1+\nu_b)U(c,Q(e)) - \nu_b V^D(y)\Big] \\
&+ \Big[(1+\nu_l)[y-c] - \nu_l Z(y)\Big] \\
&+ \int \Big[\frac{1+\nu_l}{1+r} FV(y',x'(y')) - x\varphi(y'|y)v_{Q(e)}(y')\Big] Q(y'|y,e)(\mathrm{d}y')\Big\} \\
\text{s.t.} \quad x'(y') = &\left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varphi(y'|y)}{1+\nu_l}\right]\eta x \quad \text{and} \quad \varphi(y'|y) = \varrho \frac{\partial_e Q(y'|y,e)}{Q(y'|y,e)}.
\end{aligned}$$

6 Quantitative Analysis

We first calibrate the Fund's outside option for the Euro Area stressed countries. We then compare the outcome of the different Fund contracts in terms of business cycles properties and welfare.

6.1 The Quantitative Fund

For the quantitative model, we expand the Fund contract in several dimensions. We expose the effect of these changes in the Fund under flexible MH. The other Fund contracts are derived in the Appendix. First, the borrower can produce goods using a decreasing-returns labour technology $y = \theta f(n)$, where $f'(n) > 0$, $f''(n) < 0$, $n \in [0,1]$ denotes labor and θ is a productivity shock. The shock is composed of two parts

$$\theta \equiv \zeta + \varsigma(\zeta)g,$$

where $\varsigma(\zeta)$ denotes the standard error of θ conditional on ζ. The shock ζ follows a Markov process with compact support $P \subseteq \mathbb{R}^+$ and transition function $\pi^\zeta(\zeta'|\zeta)$. For the shock g, the government can generate any *distribution* with compact support $K \subseteq \mathbb{R}^+$. We denote by π the distribution of g' conditional on ζ' and $\boldsymbol{\pi}^g = \{\zeta' \in P : \pi \in \mathcal{M}\}$ the vector of all such conditional distributions. While the two shocks ζ' and g' are directly contractible, the vector of conditional distributions $\boldsymbol{\pi}^g$ is not. We denote the state at the beginning of a period to be $s \equiv \{\zeta, g\}$.

Second, we assume a more realistic outside option which corresponds to the autarky value in a Incomplete Market economy with Defaultable (IMD) debt,[10]

$$
\begin{aligned}
V^D(s) = &\max_{n,\boldsymbol{\pi}^g} \Big\{ U(\theta^d f(n), n, \boldsymbol{\pi}^g) \\
&\qquad + \beta \int \int \big[(1-\lambda)V^D(s') + \lambda J(s', 0)\big] \pi(\mathrm{d}g') \pi^{\zeta}(\zeta'|\zeta)(\mathrm{d}\zeta') \Big\},
\end{aligned} \tag{20}
$$

where $\theta^d \leq \theta$ contains the penalty for defaulting and $\lambda \geq 0$ is the probability to re-access the private bond market. Furthermore, $J(\cdot)$ corresponds to the value of reintegrating the private bond market without the Fund. More precisely, $J(s,b) = \max_{D\in\{0,1\}}\{(1-D)V^P(s,b) + DV^D(s)\}$, with

$$
\begin{aligned}
V^P(s,b) = &\max_{\{c,n,\boldsymbol{\pi}^g,b'\}} \Big\{ U(c,n,\boldsymbol{\pi}^g) + \beta \int \int \big[J(s',b')\big] \pi(\mathrm{d}g')\pi^{\zeta}(\zeta'|\zeta)(\mathrm{d}\zeta') \Big\} \\
&\text{s.t. } c + q(s,b')(b' - \delta b) \leq \theta f(n) + (1 - \delta + \delta\kappa) b.
\end{aligned} \tag{21}
$$

In the private bond market, the government can borrow long-term defaultable bonds, b', at a unit price of $q_p(s,b')$. A fraction $1-\delta$ of each bond matures today and the remaining fraction δ is rolled-over and pays a coupon κ. Private lenders are competitive and the price of one unit of private bond is given by $q(s,b') = \frac{1}{1+r} \int \int (1 - D(s',b'))[1 - \delta + \delta\kappa + \delta q(s',b'')] \pi(\mathrm{d}g')\pi^{\zeta}(\zeta'|\zeta)(\mathrm{d}\zeta')$ where $D(\cdot)$ is the default policy taking value one in case of default and zero otherwise.

In this extended environment, the Fund's contract in sequential form is given by

$$
\begin{aligned}
\max_{\{c(s^t),n(s^t),\boldsymbol{\pi}^g_{t+1}\}} \ & \mathbb{E}_0 \left[\alpha_{b,0} \sum_{t=0}^{\infty} \beta^t U(c(s^t), n(s^t), \boldsymbol{\pi}^g_{t+1}) + \alpha_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \big[\theta_t f(n(s^t)) - c(s^t)\big] \right] \\
\text{s.t.} \quad & \mathbb{E}_t \left[\sum_{j=t}^{\infty} \beta^{j-t} U(c(s^j), n(s^j), \boldsymbol{\pi}^g_{j+1}) \right] \geq V^D(s_t) \\
& \mathbb{E}_t \left[\sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \big(\theta_j f(n(s^j)) - c(s^j)\big) \right] \geq Z(s_t) \\
& v_{\pi_{t+1}(\zeta^{t+1})}(g^{t+1}) = \beta \left(V^b(s^{t+1}) - V^b(\{\zeta^{t+1}, \underline{y}\}) \right).
\end{aligned}
$$

As one can see the different constraints are easily adapted to the extensions we consider.

[10] See Aguiar et al. (2009), Arellano (2008), Chatterjee and Eyigungor (2012) and Aguiar and Amador (2021).

The Fund's contract in recursive form is then

$$FV(x,s) = \text{SP} \min_{\{\nu_b,\nu_l,\boldsymbol{\varrho}\}} \max_{\{c,n,\boldsymbol{\pi}^g\}} \Bigg\{ x\Big[(1+\nu_b)U(c,n,\boldsymbol{\pi}^\zeta) - \nu_b V^D(s)\Big] \tag{22}$$

$$+ \Big[(1+\nu_l)[\theta f(n) - c] - \nu_l Z(s)\Big]$$

$$+ \int\int \left[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - x\varrho(s')\left(v_{\pi^g(\zeta')}(g') + V_0^b(x'(\{\zeta',\underline{y}\}),\zeta')\right)\right] \pi^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta')\Bigg\}$$

$$\text{s.t.} \quad x'(s') \equiv \overline{x}'(s) + \hat{x}'(s') = \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varrho(s')}{1+\nu_l}\right]\eta x, \tag{23}$$

where $\boldsymbol{\varrho} \equiv \{\zeta' \in P, g' \in K : \varrho(\{\zeta',g'\})\}$ corresponds to the vector of multipliers attached to the IC constraints.

6.2 Calibration

Following Ábrahám et al. (2025), we calibrate the IMD economy for the Euro Area stressed countries during the euro crisis (i.e. Portugal, Italy, Greece and Spain) between 1980 and 2019. The model period is assumed to be one year. Table 1 lists all the parameters in the model. The Appendix contains more information about the data.

The utility of the borrower is additively separable in consumption, leisure and effort. In particular, we assume that $u(c) = \frac{c^{1-\sigma_c}-1}{1-\sigma_c}$ and $h(1-n) = \gamma\frac{(1-n)^{1-\sigma_l}-1}{1-\sigma_l}$. For the canonical MH, we consider $v(e) = \omega_1 e^2$, for the flexible MH $v(\pi) = \frac{\omega_2}{2}[\int(g-\underline{y})\pi(\mathrm{d}g)]^2$ and for the restricted flexible MH $v(\pi) = \frac{\omega_3}{2}[\int(g-\underline{y})\pi(\mathrm{d}g)]^2$ so that the second derivative is linear in all cases. While the value of σ_c follows the standard in the literature, we choose γ and σ_l to match the average and relative volatility of n. We explain how the parameters $(\omega_1,\omega_2,\omega_3)$ are determined when we expose the labor productivity shock estimation.

The parameters of the long term bond (δ,κ) are set to match the average maturity and the average coupon rate (coupon payment to debt ratio) of debt, respectively. After a default, the borrower faces exclusion for a random number of periods. The probability of market re-access is the one of Chatterjee and Eyigungor (2012). If a borrower defaults, it is also subject to an asymmetric default penalty $\theta^d = \min\{\theta, O(\theta)\}$ where $O(\theta) = (1-\psi)\underline{\theta} + \psi(\overline{\theta}+\theta)$.[11] The parameter ψ is chosen to match the average spread in the data. The discount factor to β is set to match the average debt ratio. The risk free interest rate r is equal to the average short-term real interest rate of Germany after the introduction of the euro from 2000 to 2019.

[11] We adopt a different cost than Arellano (2008) who assumes $O(\theta) = O = \psi\mathbb{E}\theta$. The reason is that the specification of Arellano (2008) does not guarantee that the strict inequality in Assumption **??** holds under flexible MH.

Table 1: Parameters

Parameter	Value	Description	Targeted Moment
A. Literature			
σ_c	1	Risk aversion	
Z	0	Fund mutualization	
B. Data			
r	0.0198	Risk-free rate	Average German real short-term rate
δ	0.814	Bond maturity	Average bond maturity
κ	0.076	Bond coupon rate	Average bond coupon rate
a	0.5696	Labor share	Average labor share
λ	0.1538	Market re-access probability	Average exclusion
C. Model			
β	0.9375	Discount factor	Average b/y
σ_l	0.17	Labor elasticity	Average n
γ	1.49	Leisure utility weight	Relative volatility n
ω_1	0.17	Effort disutility weight	$\mathbb{E}\varpi(e) = 0.5$
ω_2	9.7	Effort disutility weight	Average disutility of effort
ω_3	31.3	Effort disutility weight	Average disutility of effort
ψ	0.163	Output default cost	Average spread
ϵ	0.0001	Utility shock variance	Convergence

Following Ábrahám et al. (2025), Liu et al. (2020) and Callegari et al. (2023), the participation constraint of the Fund is set to $Z = 0$, implying no expected permanent transfers between the borrower and the Fund at any time or state. In other words, the Fund is not build on an assumption of solidarity which would require permanent transfers.

Regarding the production technology, we assume that $f(n) = n^a$ with the labor share a equating the average labor share across the Euro Area stressed countries. We verify that the Fund's value is concave *ex post*. The log of labor productivity, $\log \theta$, is assumed to be a Markov regime switching (MRS) AR(1) process. We fit the labor productivity $\log(\theta_{i,t})$ of the four countries to the following panel MRS AR(1) model

$$\log(\theta_{i,t}) = (1 - \rho(\zeta_{i,t}))m(\zeta_{i,t}) + \rho(\zeta_{i,t}) \log(\theta_{i,t-1}) + \varsigma(\zeta_{i,t})\epsilon_{i,t}, \tag{24}$$

where $\zeta_{i,t} \in \{1, \dots, R\}$ denotes the regime of country i at time t, $\rho(\zeta_{i,t})$, $m(\zeta_{i,t})$, $\varsigma(\zeta_{i,t})$ are the regime-specific autocorrelation, mean and standard error of the process, respectively, and $\epsilon_{i,t}$ follows an i.i.d. standard normal distribution. Given this, we

Table 2: Labor Productivity Process

	$m(\zeta)$	$\rho(\zeta)$	$\varsigma(\zeta)$	$\pi^{\zeta}(\zeta'\|\zeta)$	$\zeta' = 1$	$\zeta' = 2$
$\zeta = 1$	6.62	0.93	0.06	$\zeta = 1$	0.91	0.09
$\zeta = 2$	6.68	0.82	0.17	$\zeta = 2$	0.12	0.88

Note: The variable $\zeta \in \{1, 2\}$ denotes the regime, $\pi^{\zeta}(\zeta'|\zeta)$ corresponds to the regime transition matrix, $\rho(\zeta)$ is the regime-specific autocorrelation, $m(\zeta)$ is the regime-specific mean and $\varsigma(\zeta)$ is the regime-specific standard error of the process.

can write $\theta_{i,t} = \zeta_{i,t} + \varsigma(\zeta_{i,t})g_{i,t}$. The country specific regime $\zeta_{i,t}$ is independent in the cross-section, and follows a Markov chain over time, with an $R \times R$ regime transition matrix π^{ζ}. Since our model does not have any capital accumulation, we use the time series for the labor productivity $\theta_{i,t}$ for the four Euro Area stressed countries. The estimated parameters of the MRS are displayed in Table 2 with $R = 2$. We further discretize the shock process using the method of Liu (2017) with 20 grid points for each regime leading to a total of 40 labor productivity states θ. We then split θ between ζ and g by setting $\underline{y} = 0$ given the estimated standard error $\varsigma(\zeta)$.

The above estimation enables us to retrieve Q. Recall that $Q = \varpi(e)Q_L + (1 - \varpi(e))Q_H$ for $Q_L, Q_H \in \mathcal{Q}$. Effort affects the probability distribution over next period's realisation of y'. Knowing Q, we create Q_H using a modified version of the mass transfer algorithm in Østerdal (2010).[12] We then retrieve $Q_L = (Q - (1 - \varpi(e))Q_H)/\varpi(e)$. To facilite to computation of Q_H and Q_L, we set $\varpi(e) = 0.5$ and choose ω_1 accordingly. Regarding the exact functional form, we set $\varpi(e) = (e-1)^2$ which implies simple expressions for $\frac{\partial Q(g'|g,e)}{\partial e}$ and $\frac{\partial^2 Q(g'|g,e)}{\partial e^2}$ as follows:

$$
\begin{aligned}
\frac{\partial Q(g'|g,e)}{\partial e} &= -\varpi'(e)\big[Q_L(g'|g) - Q_H(g'|g)\big] = 2(1-e)\big[Q_L(g'|g) - Q_H(g'|g)\big] \\
\frac{\partial^2 Q(g'|g,e)}{\partial e^2} &= -\varpi''(e)\big[Q_L(g'|g) - Q_H(g'|g)\big] = -2\big[Q_L(g'|g) - Q_H(g'|g)\big].
\end{aligned}
$$

Under this functional forms Assumption 3 is satisfied. We finally select (ω_2, ω_3) such that the borrower incurs *ex post* the same average disutility of effort in all IMD economies.

[12]More precisely, we split Q into R^2 sub-transition matrices for each regime, say $Q_{i,j}$ for $i, j \in \{1, \ldots, R\}$. For each $Q_{i,j}$, we generate $Q_{H,i,j}$ by shifting the probability mass from below the main diagonal of $Q_{i,j}$ to above the main diagonal. This makes high values of θ more likely to happen. We then put all $Q_{H,i,j}$ back together which gives us Q_H. The transfer of probability mass for each sub-matrix is the maximal transfer such that $(Q - (1 - \varpi(e))Q_H)/\varpi(e) = Q_L$ is a well-defined transition matrix.

As noted by Chatterjee and Eyigungor (2012), the computation of the IMD economy with long-term debt requires some form of randomization. Building on the most recent literature, we introduce a utility shock to the default choice which follows a Type 1 Extreme Value (i.e. Gumbel) distribution with a scale parameter ϵ.[13] We do not give any structural meaning to this shock. It is a pure randomization device aimed for convergence and accordingly we pick the smallest possible value of ϵ leading to convergence.

6.3 Outcome and comparison

Table 3 depicts the outcome of the calibration. We first discuss the IMD and the Fund economies separately before comparing them.

Regarding the IMD economies, the IMD under canonical and restricted flexible MH are very similar. They both offer predictions that are very close to the data. In opposition, the IMD under flexible MH is very different. The default rate is zero and the depicted relative volatilities are below the other two IMD economies. More importantly, the debt ratio is almost 4 times higher than the targeted one. Plus, the primary surplus is perfectly counter-cyclical, while it is pro-cyclical in the data. It therefore seems that the IMD economy under flexible MH is very much at odd with the data unlike the other two IMD economies.

Regarding the Fund economies, a similar argument applies. While the Fund under canonical, restricted flexible and back-loaded MH are very close from each other, the Fund under flexible MH generates very different moments. In particular, the Fund under flexible MH records a large primary surplus and high working hours with a very low relative volatility of consumption, while the other two Fund contracts generate lower primary surpluses and working hours on average and more volatile consumption relative to output. Consumption, labor and the primary surplus are highly pro-cyclical in all Fund contracts but only perfectly so under flexible MH.

Comparing the IMD and the Fund economies, there are substantial differences. Under the canonical and the restricted flexible MH, the Fund reduces the volatility of consumption, labor and primary surplus. These variables also correlate more with output in the Fund. Regarding the flexible MH, the primary surplus and labor are perfectly pro-cyclical in the Fund and perfectly counter-cyclical in the IMD economy. Moreover, the Fund almost annihilates the relative volatility of consumption, while the IMD economy generates a relative volatility close to 1.

[13] See Mihalache (2020), Dvorkin et al. (2021), Mateos-Planas et al. (2022) and Mateos-Planas et al. (2023).

Table 3: Results

Variables	Targeted	Data	IMD Economy			Fund Economy			
			CMH	FMH	RMH	CMH	FMH	RMH	BMH
A. First moments									
b'/y (%)	$\times$	85.70	82.33	330.70	96.56	-	-	-	-
n (%)	$\times$	36.09	36.11	38.24	36.26	37.53	40.45	38.03	36.56
e		-	0.29	-	1.00	0.19	-	0.99	0.18
$(y-c)/y$ (%)		-0.76	1.38	7.89	1.64	0.74	2.51	0.97	0.42
Spread (%)	$\times$	2.20	2.26	0.00	1.88	-	-	-	-
B. Second moments									
std(c)/std(y)		1.00	1.31	1.09	1.29	0.54	0.06	0.35	0.31
std(n)/std(y)	$\times$	0.99	0.97	0.08	1.05	0.51	0.84	0.73	0.77
std($(y-c)/y$)/std(y)		1.03	1.11	0.08	1.20	0.06	0.18	0.09	0.09
std(spread)		1.20	0.92	0.00	0.67	-	-	-	-
corr(c, y)		0.79	0.61	1.00	0.52	0.91	1.00	0.71	0.62
corr(n, y)		0.66	0.19	-1.00	0.28	0.92	1.00	0.95	0.96
corr($(y-c)/y, y$)		0.17	0.18	-1.00	0.27	0.91	1.00	0.96	0.95
corr(spread, y)		-0.18	-0.35	0.00	-0.28	-	-	-	-

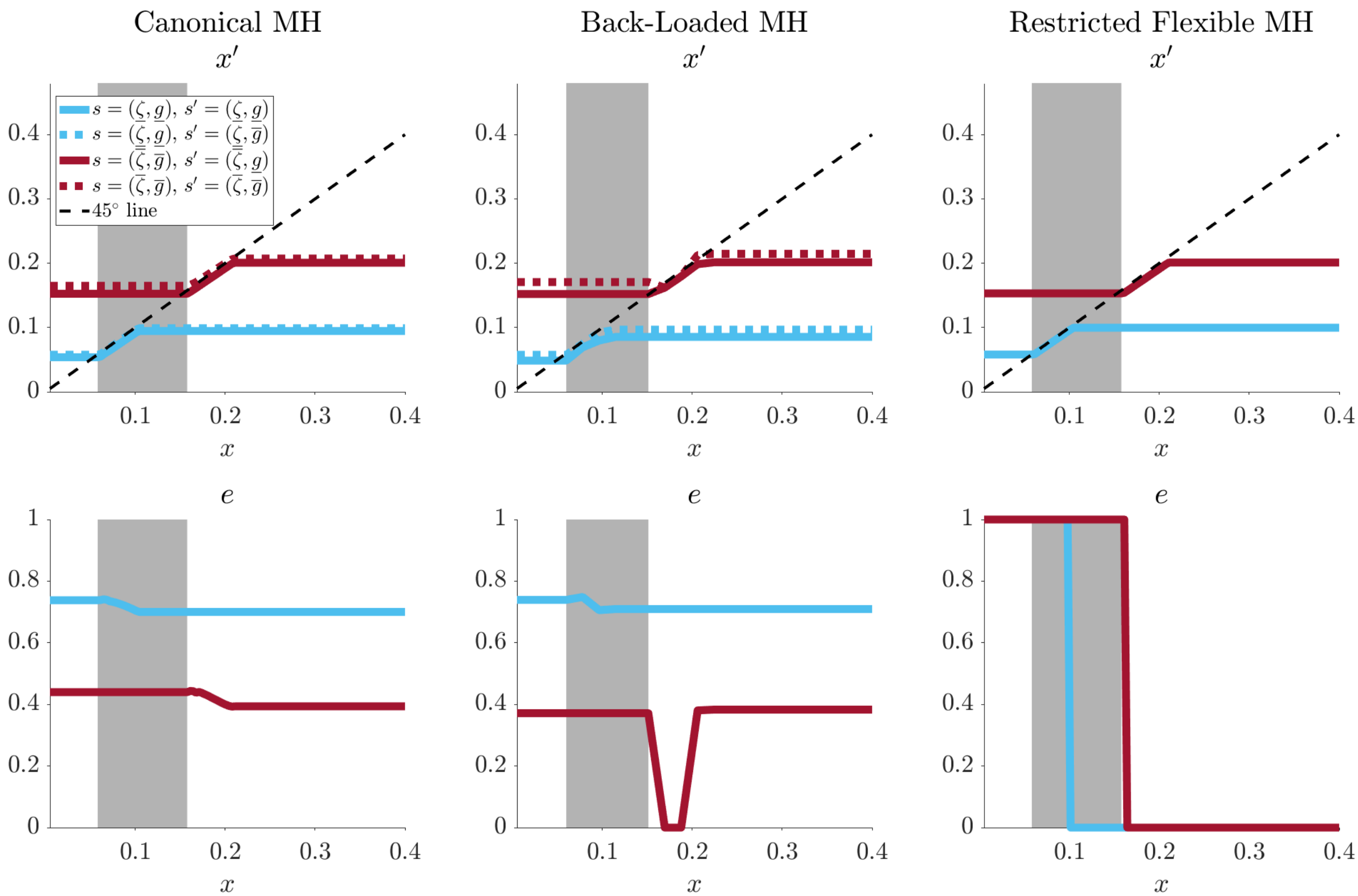
Note: CMH stands for Canonical Moral Hazard, BMH for Back-Loaded Moral Hazard, RMH for Restricted Flexible Moral Hazard and FMH for Flexible Moral Hazard. See the Appendix contains more information about the data.

6.4 Steady state analysis

This subsection focuses on the Fund's allocation in steady state. We first study the main policy functions before simulating the different economies and analyzing welfare.

Figures 1 and 2 depict the main policy functions as a function of the relative Pareto weight x. The red lines relate to the highest value of θ (i.e. $s = \{\overline{\zeta}, \overline{g}\}$) and the blue lines to the lowest value of θ (i.e. $s = \{\underline{\zeta}, \underline{y}\}$). The dotted lines relate to the highest value of y' (i.e. $\overline{g}$) and the solid lines to the lowest value of y' (i.e. $\underline{y}$). The two figures also represent the ergodic set of the relative Pareto weights for the different Fund contracts in gray. The ergodic set gives the steady state of the Fund contract in which we conduct simulations.

In each specification, the horizontal line on the left hand side is determined by the



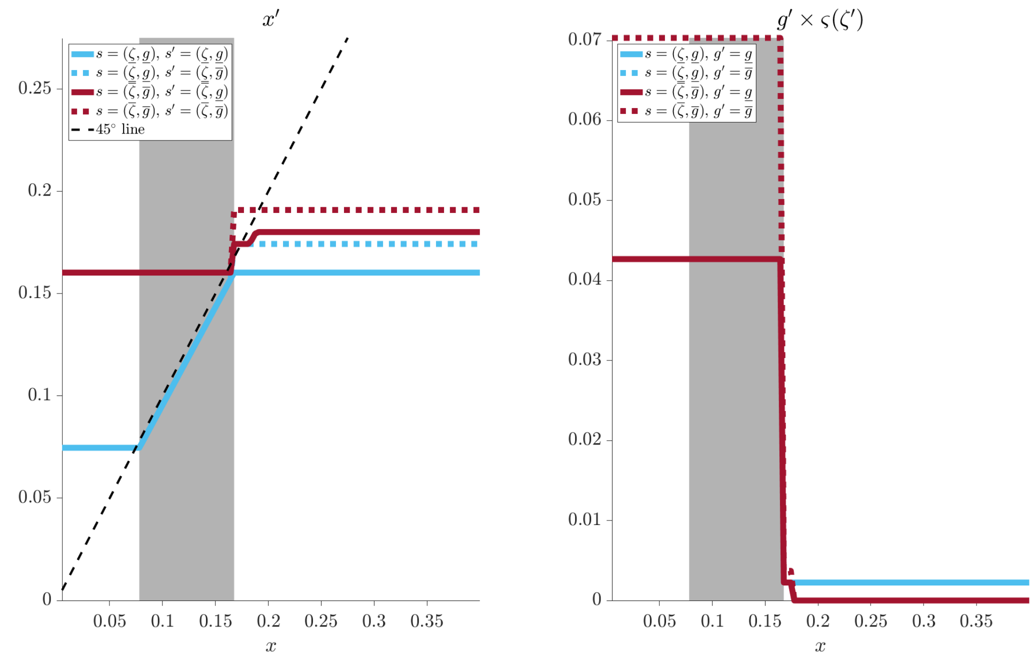
Note: The figure depicts the main policy functions as a function of the relative Pareto weight x. The red lines relate to the highest value of θ and the blue lines to the lowest value of θ. The dotted lines relate to the highest value of y' and the solid lines to the lowest value of y'. The gray region corresponds to the ergodic set.

Figure 1: Fund policies under Canonical, Back-Loaded and Restricted Flexible MH

borrower's binding LE constraint, while the horizontal line on the right hand side is determined by the Fund's binding LE constraint. The line rejoining both horizontal lines is determined by the allocation when none of the LE constraints binds.

Looking at Figure 1, we observe little differences between the Funds under canonical, back-loaded and restricted flexible MH. The only exception is that the borrower always sets $e = 1$ in the restricted flexible MH, while the effort is interior in the other two MH regimes.[14] This is because $\frac{\partial Q(g'|g,e)}{\partial e} = 0$ when $e = 1$ leading to the IC constraint (19) to hold. This also implies that the dotted and solid lines are aligned for x'. Note also

[14]In the back-loaded MH, effort is close to but not exactly zero outside the ergodic set for high values of θ.



Note: The figure depicts the main policy functions as a function of the relative Pareto weight x. The red lines relate to the highest value of θ and the blue lines to the lowest value of θ. The dotted lines relate to the highest value of y' and the solid lines to the lowest value of y'. The gray region corresponds to the ergodic set.

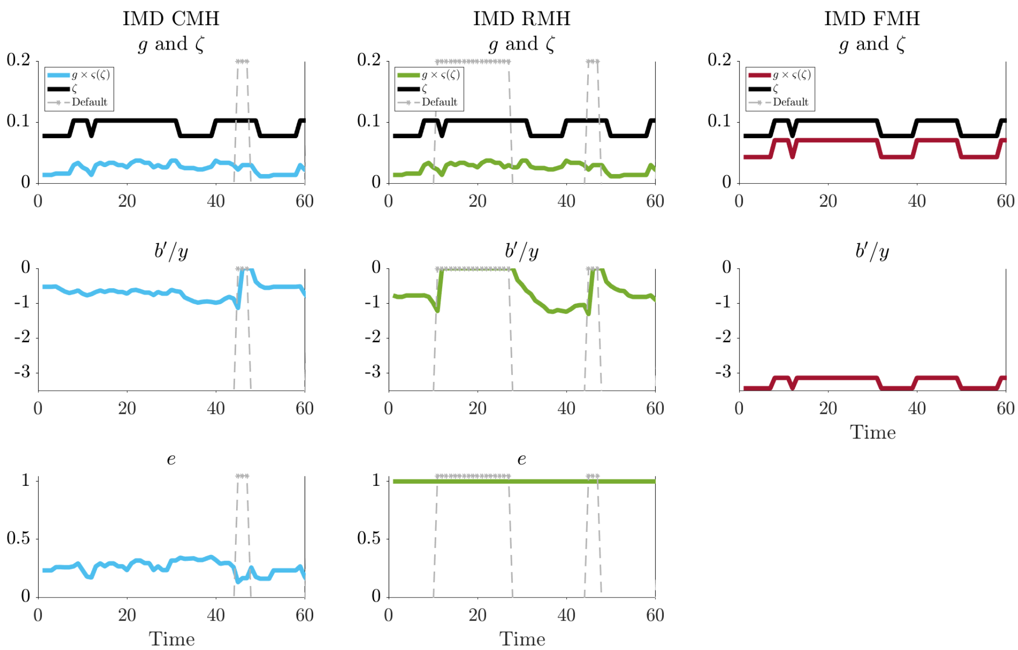
Figure 2: Fund policies under Flexible MH

that there are some non-monotonicity in the level of effort in the back-loaded MH. As noted by Ábrahám et al. (2025) this is due to the Fund's binding LE constraint.

Looking at Figure 2, the borrower chooses a relatively high y' when the the Fund's LE constraint does not bind. However, it sets $y' = 0$ in most states in which this constraint binds. The reason is that the spread in the borrower's and the Fund's value between the different choices of y' is too narrow to sustain high values of y'. This however happens outside of the steady state.

Contrasting Figures 1 and 2, we find some differences in the law of motion of the relative Pareto weights between the flexible MH and the other regimes. Moreover, the ergodic set in the Fund with flexible MH is associated with a range of relative Paeto weights which is higher than the ergodic set of the Fund under other MH regimes. As a result, borrower in the Fund under flexible MH enjoys a higher value than in the other Fund contracts as one will see later.

To better compare the different IMD and Fund economies, Figures 3 and 4 depict a simulation path under the same sequence of ζ in steady state. Except for the economies under flexible MH, we also consider the same sequence of g for illustrative purposes.



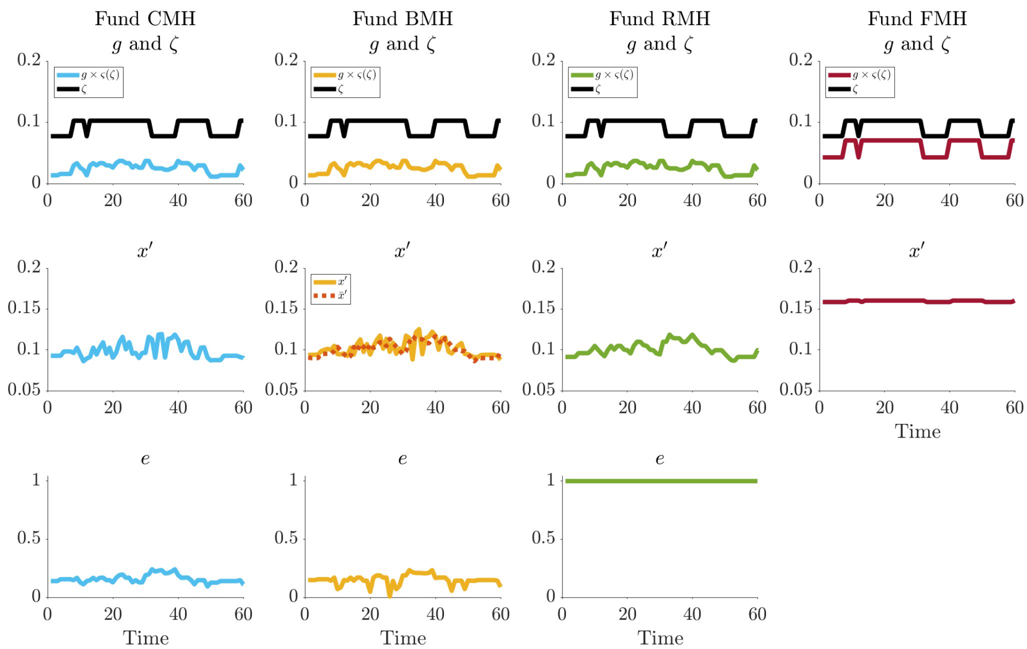
Note: The figure depicts a simulated path of debt and effort in steady state in the different IMD economies.

Figure 3: Simulation – IMD economies

In general, because the different economies exert varying levels of effort, the sequence of g differs.

Figure 3 depicts a simulated path of debt and effort in steady state in the different IMD economies. The gray dotted lines represent the occurrence of defaults. We observe that the different IMD economies have distinct dynamics of indebtedness. Under flexible MH, the borrower can sustain a large amount of debt without ever defaulting. Under restricted flexible MH, the economy accumulates more debt than under canonical MH. This however translates into a higher frequency of default in the period considered. Regarding effort, the borrower fixes $e = 1$ under restricted flexible MH as noted previously. Under canonical MH, effort is always strictly less than 1. Under flexible MH, the chosen level of g perfectly tracks the path of ζ.

Figure 4 depicts a simulated path like Figure 3 but for the different Fund economies. In terms of relative Pareto weights, the Fund under back-loaded MH provides an interesting case. When looking at the latent Pareto weight x' (yolid yellow line), the depicted path follows the one of the relative Pareto weight under canonical MH. However, the main relative Pareto weight $\bar{x}'$ (dotted orange line) follows very closely the relative Pareto weight in the restricted flexible MH. This is because with $e = 1$ and the functional form of $\varpi(e)$, $\varphi(y'|y) = 0$ in the restricted flexible MH. In opposition, the



Note: The figure depicts a simulated path of debt and effort in steady state in the different Fund economies.

Figure 4: Simulation – Fund economies

path of the relative Pareto weight under flexible MH remains flat. In terms of effort, we have $e = 1$ under restricted flexible MH, while, under flexible MH, the chosen level of g tracks the path of ζ similar to the IMD economies.

We end this section with a welfare analysis in steady state. Welfare gains are computed for the borrower as a percent of consumption-equivalent changes. Denoting the value of the borrower in the benchmark case by $V^b(\theta)$ and in the alternative case by $\ddot{V}^b(\theta)$, the gains are given by $(\exp\left[(\ddot{V}^b(\theta) - V^b(\theta))(1-\beta)\right] - 1) \times 100$ under the assumed functional form of the utility function. For the lenders, we simply compute $\ddot{V}^l(\theta) - V^l(\theta)$ as a proxy of welfare gains. Table 4 depicts the welfare gains and is made of two parts. The upper part of the table presents the gains for the different IMD economies with respect to the IMD economy under canonical MH (i.e. the benchmark case) for the different MH regimes, whereas the lower part compares the different Fund contracts with respect to the Fund contract under canonical MH (i.e. the benchmark case).

When comparing the different IMD economies, we observe welfare gains for the borrower. The IMD economy with flexible MH Pareto dominates all the other alternatives for the borrower. We also note that the borrower is better off in the IMD economy under restricted flexible than under canonical MH. The wedge in the values of the lenders is however negligible in all cases.

Table 4: Welfare Gains

IMD vs. CMH IMD

State	Borrower		Lenders/Fund	
	FMH	RMH	FMH	RMH
$\underline{\theta}$	78.17	7.07	0.00	0.00
$\overline{\theta}$	34.04	3.70	0.00	0.00

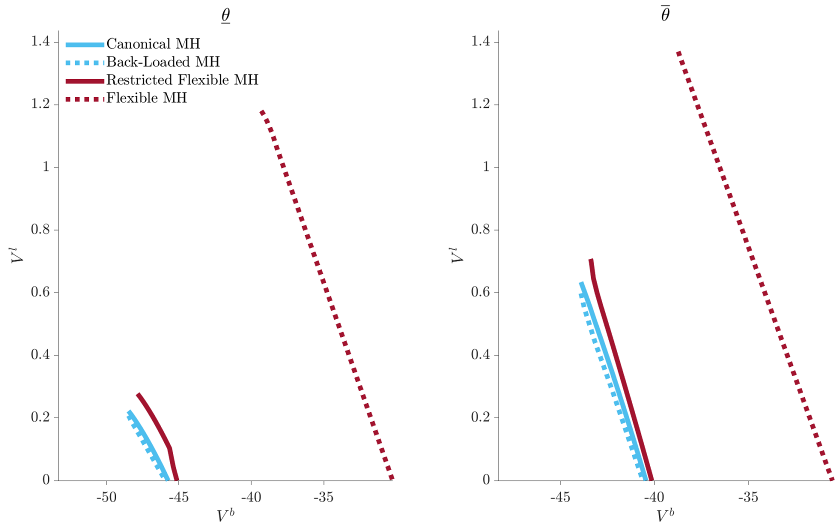
Fund vs. CMH Fund

State	Borrower			Fund		
	FMH	RMH	BMH	FMH	RMH	BMH
$\underline{\theta}$	82.09	9.69	1.12	0.97	0.07	-0.12
$\overline{\theta}$	29.65	6.15	3.67	0.67	0.11	-0.19

Note: CMH stands for Canonical Moral Hazard, BMH for Back-Loaded Moral Hazard, RMH for Restricted Flexible Moral Hazard and FMH for Flexible Moral Hazard. The borrower's welfare gains for a specific θ correspond to $(\exp\left[(\ddot{V}^b(\theta)-V^b(\theta))(1-\beta)\right]-1)\times 100$ where $\ddot{V}^b$ and V^b are the values of the borrower in the benchmark and the alternative case, respectively. For the lenders, welfare gains are simply given by $\ddot{V}^l(\theta)-V^l(\theta)$.

Regarding the comparison across Fund contracts, we see that the Fund under flexible and restricted flexible MH Pareto dominate the Fund contract under canonical MH. Under the flexible MH, the Fund almost completely eliminates the volatility of consumption. Under the restricted flexible MH, consumption is also less volatile than under the canonical MH and effort is larger. Under the back-loaded MH, the outcome is different. While the borrower is better off than in the canonical MH, the Fund is worse off. This should not come as a surprise as the borrower benefits from a better consumption smoothing mechanism which comes at the cost of the Fund.

To complement the computation of welfare gains in steady state, Figure 5 depicts the Pareto frontier of the different Fund contracts. Such frontiers span the entire state space and are not restricted to the ergodic set. We see that the Fund contract under canonical MH is Pareto dominated by all the other contracts except for the Fund under the back-loaded MH. The most efficient contract is the one under flexible MH followed by the one under restricted flexible MH as the welfare gains already highlighted.



Note: The figure depicts the Pareto frontier of the different Fund contract. The panel on the left is for $\underline{\theta}$ and the panel on the right is for $\overline{\theta}$.

Figure 5: Pareto Frontiers

7 Conclusion

From the perspective of economic theory, since the pioneer work of Prescott and Townsend (1984), it is understood that under appropriate convexity assumptions moral hazard (and adverse selection) problems can be incorporated as problems of efficiently assigning resources subject to technological and feasibility constraints, by introducing Incentive Compatibility (IC) constraints in parallel to other constraints. Furthermore, it is also understood that under these, and other standard assumptions, the corresponding competitive equilibrium exists and the First and Second Welfare Theorems are satisfied for constrained-efficient allocations. Extensive follow up work has extended these results to dynamic economies – e.g. with debt or other financial assets, etc. However, all this work has built on – what we call – the *canonical* framework, and not much work has been done in studying different forms of implementation. In fact, from the applied perspective – say, of official lenders – IC design has had almost no impact and the focus has been on the design of verifiable conditions, signaling the improvement of a risk profile.

In this paper, we have widen the scope of dynamic IC design by: extending the *flexible moral hazard* approach to dynamic contracts, in particular to recursive contracts with limited enforcement constraints; confronting it with the *canonical* approach, and bringing them closer with the *restricted flexible* and *back-loaded* designs. The latter being close to the official lending programs, which often become a sequence of short-term programs, which, as we show, can be a way to implement ICs.

Our quantitative results open a new venue for the design of official lending programs, since in assessing the risk-profile of a country the first question that arises is: what is its capacity to choose a better risk distribution? and how costly would that be? Then, design the ICs accordingly. While, as we find, unconstrained flexible MH is counterfactual in the context of sovereign debt, forms of constrained flexible moral hazard are likely to be factual and, therefore, its ICs implementable. In fact, conditional reforms to prevent pandemics or natural disasters are choices of distributions.

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Online Appendix (Not For Publication)

A Proofs

A.1 Preliminary lemmas

We first prove a few preliminary lemmas before proving the propositions and lemmas located in the main text. We start with the characteristics of the borrower's and the Fund's value.

Lemma A.1. *Under Assumption 1, in the Fund contract under flexible MH:*

1. *When none of the LE constraints binds, $x'(y, x, y')$ and $c(y, x)$ are strictly increasing in x and $\pi(y, x)$ is strictly increasing in first-order stochastic dominance in x, $V^b(y, x)$ is strictly increasing and strictly concave in x and $V^l(y, x)$ is strictly decreasing and concave in x.*
2. *When one of the LE constraints binds, $x'(y, x, y')$, $c(y, x)$, $\pi(y, x)$, $V^b(y, x)$ and $V^l(y, x)$ are all constant in x.*
3. *$V^b(x'(y, x, y'), y')$ is strictly increasing and strictly concave in y' and $V^l(x'(y, x, y'), y')$ is strictoly increasing and concave in y'.*

Proof. For the flexible MH, recall that

$$FV(y, x) = xV^b(y, x) + V^l(y, x) \text{ with}$$
$$V^l(y, x) = y + \frac{1}{1 + r} \int \left[V^l(y', x'(y'))\right] \pi(y')(\mathrm{d}y'),$$
$$V^b(y, x) = U(c, \pi) + \beta \int \left[V^b(y', x'(y'))\right] \pi(y')(\mathrm{d}y').$$

We first show that

$$\partial_x FV(y, x) = V^b(y, x) + x\partial_x V^b(y, x) + \partial_x V^l(y, x) = V^b(y, x),$$

which implies the efficient risk-sharing property: $x\partial_x V^b(y, x) = -\partial_x V^l(y, x)$. This comes from the envelope condition stating that $\partial_x FV = V^b(y, x)$. At the same time, the decomposition in FV leads to $\partial_x FV(y, x) = Vb(y, x) + x\partial_x V^b(y, x) + \partial_x V^l(y, x)$. Combining these two equations delivers the desired result.

We second show that $x'(y, s, y')$ and $c(y, x)$ are strictly increasing in x and $y'(y, x)$ is decreasing in x when none of the LE constraints binds. For $x'(y')$, the statement directly follows from the law of motion of the relative Pareto weight in (23). For consumption, the statement follows from the first-order conditions (10). For $\pi(y,x)$, the statement follows from (4) and $V^b(y,x)$ being increasing in x as we show next.

We third show the properties of $V^b(y,x)$ and $V^l(y,x)$ when none of the LE constraints binds. By definition, when x increases the Fund gives more weight to the borrower. As a result, $V^b(y,x)$ is strictly monotone in x. Then, using our first result, $\partial_x V^l(y,x) < 0$ so that $V^l(y,x)$ is strictly decreasing in x. We show concavity in x below together with y.

We fourth show that all policies and value functions are constant when none of the LE constraints binds. This follows from the fact that the policies, value functions and multipliers are evaluated when the constraints are binding as solutions to the saddle-point problem.

We fifth show that $x'(y')$, $V^b(y', x'(y'))$ and $V^l(y', x'(y'))$ increasing in y'. For $x'(y')$, the statement directly follows from the fact that $\varrho(y') \geq \varrho(\tilde{y}') \geq 0$ for $y' \geq \tilde{y}'$ and with strict inequality when $y' > \tilde{y}'$ as shown in Lemma 1. Regarding the value function, we have seen that $FV(y,x)$ is increasing in x. Given x, a higher y means a higher surplus and therefore a higher $FV(y,x)$ and, through risk-sharing, a higher $V^b(y,x)$ and $V^l(y,x)$.

Finally to show concavity of $V^l(y,x)$ and strict concavity of $V^b(y,x)$ in (y,x), define the operators

$$\mathcal{T}V^l(y,x) = y - c(y,x) + \frac{1}{1+r}\int V^l(y', x'(x,y,y'))\pi(y,x)(\mathrm{d}y'),$$
$$\mathcal{Q}V^b(y,x) = U(c(y,x), \pi(y,x)) + \beta \int V^b(y', x'(x,y,y'))\pi(y,x)(\mathrm{d}y').$$

Consider $\tilde{y}, \ddot{y} \in Y$ and define $y_\alpha = \alpha\tilde{y} + (1-\alpha)\bar{y}$ with $\alpha \in (0,1)$. Since the constraint set is convex, we can define $c_\alpha(x) = \alpha c(\tilde{y}, x) + (1-\alpha)c(\bar{y}, x)$, $\pi_\alpha(x) = \alpha\pi(\tilde{y}, x) + (1-\alpha)\pi(\bar{y}, x)$ and $x'_\alpha(x, y') = \alpha x'(\tilde{y}, x, y') + (1-\alpha)x'(\bar{y}, x, y')$. Assuming that V^l is concave, it then holds that

$$\begin{aligned}
\mathcal{T}V^l(y_\alpha, x) \geq & y_\alpha - c_\alpha(x) + \frac{1}{1+r}\int V^l(y', x'_\alpha(x,y'))\pi_\alpha(x)(\mathrm{d}y') \\
\geq & \alpha\left[\tilde{y} - c(\tilde{y}, x) + \frac{1}{1+r}\int V^l(y', x'(x,\tilde{y},y'))\pi(\tilde{y},x)(\mathrm{d}y')\right] \\
& + (1-\alpha)\left[\ddot{y} - c(\ddot{y}, x) + \frac{1}{1+r}\int V^l(y', x'(x,\ddot{y},y'))\pi(\ddot{y},x)(\mathrm{d}y')\right] \\
& = \alpha\mathcal{T}V^l(\tilde{y}, x) + (1-\alpha)\mathcal{T}V^l(\ddot{y}, x),
\end{aligned}$$

where the first inequality comes from optimality, the second from the assumption of concavity and third from the definition of the operator $\mathcal{T}$. Hence, V^l is concave in y. The same argument can be extended to $x_\alpha = \alpha\tilde{x} + (1-\alpha)\bar{x}$ to show concavity in x

For the operator $\mathcal{Q}$, one can repeat the same argument with $y_\alpha = \alpha\tilde{y} + (1-\alpha)\bar{y}$ and $x_\alpha = \alpha\tilde{x} + (1-\alpha)\bar{x}$, respectively. The only exception is that the second inequality is strict given the strict concavity of the instantaneous utility function. Hence, V^b is strictly concave in (y, x). □

Ábrahám et al. (2025, Lemmas 1-3) provide a proof of the same lemma in the case of canonical MH. The extension to the case of restricted flexible MH is straightforward. For the back-loaded MH, we need to distinguish between the main and the latent relative Pareto weight. Other than that, the same type of argument ought to apply.

We can now show that the the saddle-point Lagrangian is also concave in π under the assumption that the second and the third Gateau derivatives of $v(\cdot)$ are non negative.

Lemma A.2. *The Lagrangian of the saddle-point Bellman equation $\mathcal{L}(y, x)$ is concave in π.*

Proof. First observe that the first Gateau derivative of the saddle-point Lagrangian is given by $\Lambda_\pi(y')$. Hence, the second Gateau derivative of the saddle-point Lagrangian is

$$-x\left(1 + \nu_b + 2\varrho(y')\right) w_\pi(i, y') - x\varrho(y') \int z_\pi(i, j, y')\pi(\mathrm{d}i),$$

where $z_\pi(i, j, y')$ denotes the third Gateau derivative of $v(\cdot)$. By Assumption 2, $\varrho(y') \geq 0$. Moreover, the second and the third derivatives are non negative implying concavity of the saddle-point Lagrangian. □

A.2 Proof of Proposition 2

Recall that

$$\begin{aligned}\Lambda_\pi(y') \equiv & x\left(1 + \nu_b + \varrho(y')\right)\left[\beta\left(V^b(y', x'(y')) - V^b(\underline{y}, x'(\underline{y}))\right) - v_{\pi(y')}(y')\right] \\ & + \frac{1+\nu_l}{1+r}\left[V^l(y', x'(y')) - V^l(\underline{y}, x'(\underline{y}))\right] - x\varrho(y') \int w_\pi(i, y')\pi(\mathrm{d}i).\end{aligned}$$

Given (4), the expression simplifies to

$$\Lambda_\pi(y') \equiv \frac{1+\nu_l}{1+r}\left[V^l(y', x'(y')) - V^l(\underline{y}, x'(\underline{y}))\right] - x\varrho(y') \int w_\pi(i, y')\pi(\mathrm{d}i).$$

As shown in Lemma A.1, the Fund's value is concave in y. This together with the assumption that the second Gateau derivative $w_\pi(i, y')$ is strictly convex makes $\Lambda_\pi(y')$ strictly concave. We can therefore apply Corollary 3 in Georgiadis et al. (2024) stating that the distribution has at most one y' in its support. As a result, we can reformulate (22) as a problem of choosing y' directly instead of π.

A.3 Proof of Proposition 1

To show existence we use the argument in the proof of Theorem 3 of Marcet and Marimon (2019) and the proof of Proposition 1 of Ábrahám et al. (2025).

Marcet and Marimon (2019) make the following necessary assumptions: A1 a well defined Markov chain process for y, A2 continuity in $\{c, \pi\}$ and measurability in y, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lenders and strict concavity for the borrower, and a strict interiority condition. Assumption A1, A2, A5 and A6 are trivially met given Assumption 1. Since feasible c and π are bounded, payoffs functions are bounded as well. Since $\phi(\cdot) \geq 0$ and $\phi'(\cdot) \in [0, 1]$, $V^D(y)$ is montone in y which ensures that A4 is met. Whether A3 is satisfied depends on the initial condition $(y_0, x_0(y_0))$. Assumption 2 ensures feasibility and that the strict interiority condition is satisfied.

Similar to Ábrahám et al. (2025), we consider a relaxed contracting problem which is the same as the original contracting problem except that (4) is replaced by a weak inequality version. More precisely,

$$\beta \left(V^b(y^{t+1}) - V^b(\underline{y}) \right) - v_{\pi_{t+1}}(y^{t+1}) \geq 0. \tag{A.1}$$

Taking the Gateau derivative of this expression leads to

$$-w_{\pi_{t+1}}(y^{t+1}, i) \leq 0,$$

where the inequality follows from Assumption 1 stating that the second derivative is non negative. As a result, (A.1) defines a convex set of feasible distribution choices. $^{I}tshouldbenotedthatTheorem3inMarcetandMarimon$ (2019)$isthe recursive, saddle-$ $point, representation corresponding to the original contract problem(22).Toobtaintherecursiveformulationo$ $statevariable.Wereliedonthethehomogeneityofdegreeonein(\mu_b, \mu_l)$ to redefine the contracting problem using x – i.e. effectively $(x, 1)$ – as a co-state variable. Given this and the fact that multipliers are uniformly bounded, the theorem applies. That is, if we define the set of of feasible Lagrange multipliers by $L = \{(\mu_b, \mu_l) \in \mathbb{R}^2_+\}$ and the set of feasible allocations by $A = \{c \in \mathbb{R}^+, \pi \in \mathcal{M}\}$, the correspondence $SP : A \times L \to A \times L$ mapping non-empty, convex, and compact sets to themselves, is non-empty, convex-valued, and upper hemicontinuous. I can therefore apply Kakutani's fixed point theorem and existence immediately follows.

Given this, we need to show that the relaxed contracting problem has the same solution as the original contracting problem. For this it suffices to show that $\varrho(y, x, y') > 0$.

Assume by contradiction that $\varrho(y,x,y')=0$, then $\Lambda_\pi(y')=0$, implies that

$$\begin{aligned}0 =& x\,(1+\nu_b)\left[\beta\left(V^b(y',x'(y'))-V^b(\underline{y},x'(\underline{y}))\right)-v_\pi(y')\right]\\&+\frac{1+\nu_l}{1+r}\left[V^l(y',x'(y'))-V^l(\underline{y},x'(\underline{y}))\right].\end{aligned}$$

Given the monotonicity of V^l, the second line is (weakly) positive meaning that the first line needs to be (weakly) negative. This contradicts (A.1).

Finally, as FV is monotone in x, constant when either of the LE constraints are binding and concave when both are slack, we can directly use the argument of Marcet and Marimon (2019) who show that the saddle point functional equation (22) is a contraction mapping. The strict concavity/convexity assumptions on u,f and v imply that the allocation is unique.

A.4 Proof of Lemma 1

From (11), when $y'=\underline{y}$, $V^l(y',x'(y'))=V^l(\underline{y},x'(\underline{y}))$ meaning that the first-order condition is satisfied only if $\varrho_\pi(y')=0$. In opposition, when $y>\underline{y}$, $V^l(y',x'(y'))>V^l(\underline{y},x'(\underline{y}))$ from Lemma A.1 meaning that the first-order condition is satisfied only if $\varrho_\pi(y')>0$.

A.5 Proof of Corollary 1

When $\eta=1$ and $\nu_l(y)=0$ in all states, the law of motion of the relative Pareto weight simplifies to the following expression

$$\mathbb{E}x'(y')\equiv\mathbb{E}\left[\bar{x}'(y)+\hat{x}'(y')\right]=\mathbb{E}\left[(1+\nu_b(y))x(y)+\varrho_\pi(y'|y)x(y)\right].$$

Since $(\nu_b(y),\varrho_\pi(y'))\geq 0$, we get that $\mathbb{E}x'(y')\geq x(y)$.

A.6 Proof of Lemma 2

Regarding the first part of the lemma, note that, given (16),

$$\int Q(y'|y,e(y))\hat{x}'(y)(\mathrm{d}y')=0,$$

since independently of effort $\int Q(y'|y,e(y))(\mathrm{d}y')=1$, hence $\int\partial_e Q(y'|y,e(y))(\mathrm{d}y')=0$. Therefore $\mathbb{E}\hat{x}'=0$ and $\mathbb{E}x'=\bar{x}'(y)$. Alternatively, the expected law of motion of x can also be expressed as

$$\mathbb{E}x'=\mathbb{E}\left[\frac{1}{u'(c')}\frac{1+\nu_l(y')}{1+\nu_b(y')}\right]=\frac{1}{u'(c)}\eta,$$

where the last equality is the *inverse Euler equation* of the recursive contract (Ábrahám et al. (2025), Lemma 4).

Regarding the second part of the lemma, note that, since the limited enforcement multipliers are either zero or at most one of the two is positive, we can have the following decomposition

$$\mathbb{E}\frac{1}{u'(c')} = \mathbb{E}\left[x'\frac{1+\nu_b(y')}{1+\nu_l(y')}\right] = \mathbb{E}x' + \mathbb{E}x'\nu_b(y) - \mathbb{E}x'\frac{\nu_l(y)}{1+\nu_l(y)},$$

where $\mathbb{E}x' = \eta x$ and, without incentive constraints, the last two terms simply denote the change in the relative Pareto weight when either the no-default or the sustainability constraints binds. However, if LE constraints satisfy the 'no-free-lunch condition', the no-default constraint is more likely to bind, while the sustainabiliy constraint is less likely to bind and, as a result, in both cases expected consumption increases.

B Quantitative Fund Contracts

In this section, we derive the quantitative version of the Fund contracts under canonical, back-loaded and restricted flexible MH.

B.1 Canonical moral hazard

We assume independence between ζ' and g'. This implies that a single shock variable, g', depends on effort. We denote the joint distribution of ζ' and g' by $\Upsilon(s'|s,e)$. Given this, the Fund contract under canonical MH in recursive form is given by

$$\begin{aligned}
\hat{FV}(x,s) = \mathrm{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ & x\Big[(1+\nu_b)\hat{U}(c,n,e) - \nu_b \hat{V}^D(s) - \varrho \hat{v}_e(e)\Big] \\
& + \big[(1+\nu_l)(\theta f(n) - c) - \nu_l Z(s)\big] + \frac{1+\nu_l}{1+r}\int\int \hat{FV}(x',s')\Upsilon(s'|s,e)(\mathrm{d}g')(\mathrm{d}\zeta')\Big\} \\
\text{s.t. } & x'(s') \equiv \bar{x}'(s) + \hat{x}'(s') = \left[\frac{1+\nu^b}{1+\nu_l} + \frac{\varphi(s'|s)}{1+\nu_l}\right]\eta x, \\
& \varphi(s'|s) = \varrho \frac{\partial_e \Upsilon(s'|s,e)}{\Upsilon(s'|s,e)}.
\end{aligned}$$

The effort policy $e(x,s)$ is determined by the first order condition of the SPFE with respect to e,

$$\begin{aligned}\hat{v}'(e(x,s)) = \beta \int\int \partial_e \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\ + \frac{1+\nu_l(x,s)}{1+\nu_b(x,s)}\frac{1}{x}\frac{1}{1+r}\int\int \partial_e \Upsilon(s'|s,e)\hat{V}^l(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\ - \frac{\varrho(x,s)}{1+\nu_b(x,s)}\left[v''(e(x,s)) - \beta\int\int \partial_e^2 \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta')\right].\end{aligned}$$

Since the IC constraint is given by

$$\hat{v}_e(e(s)) = \beta \int\int \hat{V}^b(s')\partial_e\Upsilon(s'|s,e(s))(\mathrm{d}g')(\mathrm{d}\zeta'),$$

the first-order condition simplifies to

$$\begin{aligned}\frac{1}{1+r}\int\int \partial_e \Upsilon(s'|s,e)\hat{V}^l(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\ = \vartheta(x,s)\left[v''(e(x,s)) - \beta\int\int \partial_e^2 \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta')\right],\end{aligned}$$

where $\vartheta(x,s) \equiv \frac{x\varrho(x,s)}{1+\nu_l(x,s)}$.

B.2 Back-loaded moral hazard

The Fund contract can be expressed as the solution to a sequence of sub-contracts. As a subprogram resets when one of the LE constraint binds, the start of a subprogram is such that

$$\begin{aligned}\hat{FV}(x,s) = \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ & x\big[(1+\nu_b)(u(c)+h(1-n)-\hat{v}(e)) - \nu_b \hat{V}^D(s) - \varrho v'(e)\big] \\ & + [(1+\nu_l)(\theta f(n) - c) - \nu_l Z(s)] \\ & + \frac{1+\nu_l}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x'(s'),s')\}}\hat{FV}(x'(s'),s') + (1-\mathbb{I}_{\{(x'(s'),s')\}})\overline{FV}(x'(s'),s',\bar{x}') \mid s,e\right] \Big\} \\ \text{s.t.} \quad & x'(s') = \eta x \frac{1+\nu_b+\varphi(s'|s)}{1+\nu_l}, \\ & \bar{x}' = \eta x\frac{1+\nu_b}{1+\nu_l},\end{aligned}$$

Then within the subprogram

$$\begin{aligned}\overline{FV}(x,s,\bar{x}) = \min_{\{\varrho\}} \max_{\{c,n,e\}} \Big\{ & \bar{x}\,[u(c)+h(1-n)] - x\big[\hat{v}(e) + \varrho v'(e)\big] + (\theta f(n) - c) \\ & + \frac{1}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x'(s'),s')\}}\hat{FV}(x'(s'),s') + (1-\mathbb{I}_{\{(x'(s'),s')\}})\overline{FV}(x'(s'),s',\bar{x}') \mid s,e\right] \Big\} \\ \text{s.t.} \quad & x'(s') = \eta x\left[1+\varphi(s'|s)\right], \\ & \bar{x}' = \eta\bar{x}.\end{aligned}$$

We decompose the value as

$$\begin{aligned}
&\overline{V}_1^b(\overline{x}, s) = u(c(\overline{x}, s)) + h(1 - n(\overline{x}, s)) + \beta \mathbb{E} \overline{V}_1^b(x'(s'), s'), \\
&\overline{V}_2^b(x, s) = -\hat{v}(e(x, s)) + \beta \mathbb{E} \overline{V}_2^b(x'(s'), s'),
\end{aligned}$$

then the value of the borrower is simply $\overline{V}^b(x, s, \overline{x}) = \overline{V}_1^b(\overline{x}, s) + \overline{V}_2^b(x, s)$ implying that

$$\overline{FV}^b(x, s, \overline{x}) = \overline{x}\overline{V}_1^b(\overline{x}, s) + x\overline{V}_2^b(x, s) + \overline{V}^l(x, s, \overline{x}).$$

Given this, the unique first-order condition for the effort policy is

$$\begin{aligned}
&\hat{v}'(e) = \beta \int \int \partial_e \Upsilon(s'|s, e) \left[W^b(x'(s'), s', \bar{x}') \right] (\mathrm{d}g')(\mathrm{d}\zeta') \\
&+ \frac{1 + \nu_l}{1 + \nu_b} \frac{1}{x} \frac{1}{1 + r} \int \int \partial_e \Upsilon(s'|s, e) \left[W^l(x'(s'), s', \bar{x}') \right] (\mathrm{d}g')(\mathrm{d}\zeta') \\
&- \frac{\varrho(x, s)}{1 + \nu_b} \left[\beta \int \int \partial_e^2 \Upsilon(s'|s, e) \left[\mathbb{I}_{\{(x'(s'), s')\}} \overline{V}_2^b(x'(s'), s') + (1 - \mathbb{I}_{\{(x'(s'), s')\}}) \hat{V}^b(x'(s'), s') \right] (\mathrm{d}g')(\mathrm{d}\zeta') \right. \\
&\left. + \hat{v}''(e(x, s)) \right].
\end{aligned}$$

where $W^b(x'(s'), s', \bar{x}') = \mathbb{I}_{\{(x'(s'), s')\}} \hat{V}^b(x'(s'), s') + (1 - \mathbb{I}_{\{(x'(s'), s')\}}) \overline{V}^b(x'(s'), s', \bar{x}')$ and $W^l(x'(s'), s', \bar{x}') = \mathbb{I}_{\{(x'(s'), s')\}} \hat{V}^l(x'(s'), s') + (1 - \mathbb{I}_{\{(x'(s'), s'\bar{x}')\}}) \overline{V}^l(x'(s'), s', \bar{x}')$. As before this expression can be decomposed into the IC constraint

$$\hat{v}'(e(x, s)) = \beta \int \int \partial_e \Upsilon(s'|s, e) \left[W^b(x'(s'), s', \bar{x}') \right] (\mathrm{d}g')(\mathrm{d}\zeta'),$$

which determines $e(x, s)$, and

$$\begin{aligned}
&\frac{1}{1 + r} \int \int \partial_e \Upsilon(s'|s, e) \left[W^l(x'(s'), s', \bar{x}') \right] (\mathrm{d}g')(\mathrm{d}\zeta') \\
&= \vartheta(x, s) \left[\beta \int \int \partial_e^2 \Upsilon(s'|s, e) \left[\mathbb{I}_{\{(x'(s'), s')\}} \overline{V}_2^b(x'(s'), s') + (1 - \mathbb{I}_{\{(x'(s'), s')\}}) \hat{V}^b(x'(s'), s') \right] (\mathrm{d}g')(\mathrm{d}\zeta') \right. \\
&\qquad \left. + \hat{v}''(e(x, s)) \right],
\end{aligned}$$

which determines $\vartheta(x, s) \equiv \frac{x\varrho(x,s)}{1+\nu_l}$ defined as in Section 4.

B.3 Restricted flexible moral hazard

The Fund contract in recursive form reads as follows

$$
\begin{aligned}
FV(x,s) = \mathrm{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Bigg\{ & x\Big[(1+\nu_b)U(c,n,Q(e)) - \nu_b V^D(s)\Big] \\
& + \Big[(1+\nu_l)[\theta f(n) - c] - \nu_l Z(s)\Big] \\
& + \int\int \left[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - x\varphi(s'|s)\frac{v_{Q(e)}(g')}{\pi^{\zeta}(\zeta'|\zeta)}\right] \Upsilon(s'|s,e)(\mathrm{d}g')(\mathrm{d}\zeta')\Bigg\} \\
\text{s.t.} \quad x'(s') = & \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varphi(s'|s)}{1+\nu_l}\right]\eta x \text{ and } \varphi(s'|s) = \varrho\frac{\partial_e \Upsilon(s'|s,e)}{\Upsilon(s'|s,e)}.
\end{aligned}
$$

Taking the first-order conditions with respect to effort, we obtain the following condition

$$
\begin{aligned}
0 = & \int\int \partial_e \Upsilon(s'|s,e)\left[\beta V^b(x'(s'),s') - \frac{v_{Q(e)}(g')}{\pi^{\zeta}(\zeta'|\zeta)}\right](\mathrm{d}g')(\mathrm{d}\zeta') \\
& + \frac{1+\nu_l(x,s)}{1+\nu_b(x,s)}\frac{1}{x}\frac{1}{1+r}\int\int \partial_e \Upsilon(s'|s,e)V^l(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\
& - \frac{\varrho(x,s)}{1+\nu_b(x,s)}\Bigg[\int\int \Big\{\partial_e \Upsilon(s'|s,e)\int \frac{w_{Q(e)}(z,g')}{\pi(\zeta'|\zeta)}Q(e)(\mathrm{d}z) \\
& \qquad - \partial_e^2 \Upsilon(s'|s,e)\left[\beta V^b(x'(s'),s') - \frac{v_{Q(e)}(g')}{\pi^{\zeta}(\zeta'|\zeta)}\right]\Big\}(\mathrm{d}g')(\mathrm{d}\zeta')\Bigg].
\end{aligned}
$$

The first line is the IC constraint given by

$$
\int\int \left[\beta V^b(s') - \frac{v_{Q(e)}(g')}{\pi^{\zeta}(\zeta'|\zeta)}\right]\partial_e \Upsilon(s'|s,e(s))(\mathrm{d}g')(\mathrm{d}\zeta') = 0.
$$

As a result, the above expression becomes

$$
\begin{aligned}
& \frac{1}{1+r}\int\int \partial_e \Upsilon(s'|s,e)V^l(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\
& \quad = \vartheta(x,s)\Bigg[\int\int \Big\{\partial_e \Upsilon(s'|s,e)\int \frac{w_{Q(e)}(z,g')}{\pi(\zeta'|\zeta)}Q(e)(\mathrm{d}z) \\
& \qquad - \partial_e^2 \Upsilon(s'|s,e)\left[\beta V^b(x'(s'),s') - \frac{v_{Q(e)}(g')}{\pi^{\zeta}(\zeta'|\zeta)}\right]\Big\}(\mathrm{d}g')(\mathrm{d}\zeta')\Bigg].
\end{aligned}
$$

where $\vartheta(x,s) \equiv \frac{x\varrho(x,s)}{1+\nu_l(x,s)}$ as before.

C Data

C.1 Data Sources

Table C.1 reports the source of data used for the calibration of the model. We follow the same methodology as Ábrahám et al. (2025).

Table C.1: Data Sources and Definitions

Series	Times	Sources	Unit
Output	1980–2019	AMECO (OVGD)[a]	1 billion 2015 constant euro
Consumption	1980–2019	AMECO (OCNT)[b]	1 billion 2015 constant euro
Working hours	1980–2019	AMECO (NLHT,NLHA)[c]	1 million hours
Employment	1980–2019	AMECO (NETD)	1000 persons
Government debt	1980–2019	AMECO (EDP)	end-of-year percentage of GDP
Debt service	1980–2019	AMECO (UYIGE)[d]	end-of-year percentage of GDP
Primary surplus	1980–2019	AMECO (UBLGIE)[e]	end-of-year percentage of GDP
Bond yields	1980–2019	AMECO (ILN,ISN,ISRV)[f]	percentage
Debt maturity	1980–2019	OECD, EuroStat, ESM[g]	years
Labor share	1980–2019	AMECO[h]	percentage

[a] Strings in parentheses indicate AMECO labels of data series.

[b] PWT 8.1 values for Greece in 1980–1982.

[c] Total and average working hours.

[d] AMECO for 1995–2019; European Commission General Government Data (GDD 2002) for 1980–1995.

[e] AMECO linked series for 1995–2019; European Commission General Government Data (GDD 2002) for 1980–1995.

[f] Nominal long-term yield, nominal and real short-term yield. A few missing values for Greece and Portugal replaced by Eurostat long-term government bond yields.

[g] Average across different data sources, identical to Ábrahám et al. (2025).

[h] Compensation of employees (UWCD) plus gross operating surplus (UOGD) minus gross operating surplus adjusted for imputed compensation of self-employed (UQGD), then divided by nominal GDP (UVGD).

Labor Input. For the aggregate labor input $n_{i,t}$, we use two series from AMECO, the aggregate working hours $H_{i,t}$ and the total employment $E_{i,t}$ of each country over the period 1980-2019. We calculate the normalized labor input as $n_{i,t} = H_{i,t}/(E_{i,t} \times 5200)$, assuming 100 hours of allocatable time per worker per week. However, for most of the data moment computations, we use $H_{i,t}$ directly.

Consumption. We fit the observed fiscal behavior across the selected countries, so that we use directly the data measures of household and government consumption and government primary surplus to calibrate the model.

Government. We use the general government consolidated gross debt. As noted by Bocola et al. (2019), matching the overall public debt allows a quantitative sovereign default model to better fit crisis dynamics. Regarding the risk-free rate, we take the average real short-term yield of Germany after the introduction of the euro from 2000 to

2019. Similarly, the interest rate spread corresponds to the difference with the nominal long-term yield of Germany between 2000 and 2019. We compute the coupon rate as the ratio of debt service over debt. Finally, as noted by Ábrahám et al. (2025), the information on the maturity structure of the government debt is not comprehensive for the country considered. The overall time coverage is unequal across countries: 1998–2015 for Greece, 1991–2015 for Spain, 1990–2015 for Italy, and 1995–2015 for Portugal.

C.2 Productivity Shock Estimation

We follow Ábrahám et al. (2025) and estimate the labor productivity shock using a panel Markov regime switching AR(1) based on the expectation maximization approach of Hamilton (1990).

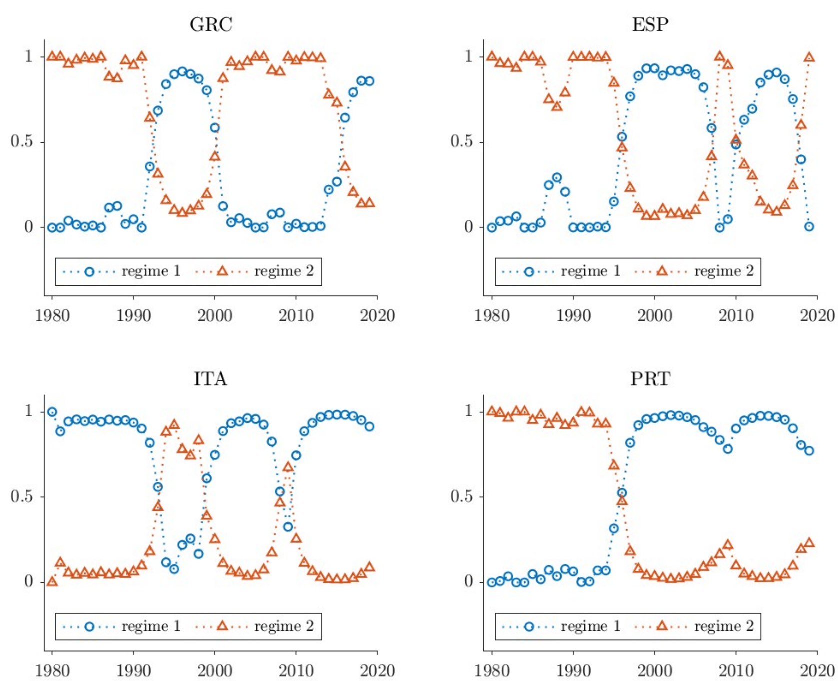

Figure C.1: Smoothed probability for each regime

Figure C.1 shows the smoothed probability for each regime across the countries included in the estimation. We consider 2 regimes: the one depicted with a circle line corresponds to a regime of low labor productivity and the one depicted with a triangle line corresponds to a regime of high labor productivity. Periods of low labor productivity are centered around the global financial crisis and the European debt crisis.

We discretize the regime switching AR(1) process with 20 grid points for each regime using the method detailed in Liu (2017).